Bachelor Thesis

AIRCRAFT PROPULSION SYSTEMS: STUDY AND SIMULATION OF A TURBOFAN

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FOREWORD

This thesis is written meaning the completion of the Bachelor Degree in Mechanical Engineering coursed at the Universitat Politècnica de Catalunya in Manresa, Spain. However, the development of the investigation has been realized in the Hochschule Aalen by means of the international mobility program Erasmus.

The task has been directed by the Doctor of Engineering Markus Merkel, professor in the Faculty of Materials and Mechanical Engineering of the Hochschule Aalen (Germany) and by Jordi Costa Vives, professor at the Escola Politècnica Superior d'Enginyeria de Manresa, from the Universitat Politècnica de Catalunya (Spain).

Along the elaboration of this project I have broaden and deepen my knowledge within the field of thermodynamics through its application in aviation. Besides, I have effectively improved my skills in the use of the computational software MATLAB Simulink and realized how useful it is in many engineering fields. Moreover and obviously, I have also learnt through the wording of the thesis how to structure and drive an investigation, from the objective attainment to the result discussion.

ACKNOWLEDGMENTS

The completion of this thesis is the outcome of the synergy of the hard work of many people. I would like to thank the directors who have supervised and advised me along the development of the project. Especially gratitude to Jordi, who has supported me no matter the distance and the difficulties entailed to it.

Thanks to M. Eng. Josef Tomas for his backing with the computational software MATLAB Simulink and his gentle solutions to all the doubts with which I approached him.

And last but not the least, I would like to thank my parents Carme and Jordi, as well as my sister Júlia for great moral support and help with proofreading the thesis.

CONTENTS

NOMENCLATURE

Symbol	Units	Description
F_T	N	Thrust force
$F_{T,net}$	N	Net thrust force
$F_{T,gross}$	N	Gross thrust force
v_o	m/s	Exiting speed
v_i	m/s	Entering speed
$\dot{m}_o$	kg/s	Exiting mass flow
$\dot{m}_i$	kg/s	Entering mass flow
P_o	Pa	Pressure at the outlet
P_i	Pa	Pressure at the inlet
A_o	m^2	Area of the specified turbofan engine's outlet
$F_{T,max}$	N	Maximum thrust force at sea-level
s	s	Engine time lag at sea-level
TSFC/SFC	kg/N·s	Thrust specific fuel consumption
M	-	Mach number
v	m/s	Speed of a particular particle or local flow velocity
θ	-	Relative temperature ratio
$T_{stagnation}$	K	Stagnation temperature
T_{ref}	K	Reference temperature
δ	-	Relative pressure ratio
$P_{stagnation}$	Pa	Stagnation Pressure
P_{ref}	Pa	Reference Pressure
σ	-	Relative density ratio
$\rho_{stagnation}$	kg/m^3	Stagnation density
ρ_{ref}	kg/m^3	Reference density
γ	-	Heat capacity ratio
c_p	J/kg·K	Specific heat capacity at constant pressure
c_v	J/kg·K	Specific heat capacity at constant volume
T_{static}	K	Static temperature
P_{static}	Pa	Static pressure
ρ_{static}	kg/m^3	Static density
τ	s	Engine time lag
T(thrust)	-	Tau(Thrust) look-up table's output
T(ndt)	-	Non-dimensional thrust look-up table's output
$F_{T,net}$	N	Net thrust force
T(tsfc)	-	Thrust specific fuel consumption look-up table's output
IU_T	-	Installed thrust to uninstalled thrust ratio

t_p	-	Throttle position
$Ø_i$	m	Inlet diameter
OPR	-	Overall Pressure Ratio
BR	-	Bypass ratio
V_i	m^3	Volume occupied by the working fluid at the *i* stage of the Brayton cycle
T_i	K	Temperature of the working fluid at the *i* stage of the Brayton cycle
P_i	K	Pressure of the working fluid at the *i* stage of the Brayton cycle
h_i	kJ/kg	Enthalpy of the working fluid at the *i* stage of the Brayton cycle
s_i	kJ/kg·K	Entropy
V_{inlet}	m^3	Volume of air entering the specified turbofan engine per second
A_{inlet}	m^2	Area of the specified turbofan engine's inlet
R_{inlet}	m	Radius of the specified turbofan engine's inlet
L	m	Longitude covered by the specified aircraft system per second
a	m/s	Speed of sound
ΔH^o_c	kJ/kg	Heat of combustion
n	Moles	Amount of substance
R	J/K·mol	Universal gas constant
R_{air}	J/kg·K	Specific gas constant for dry air
$\dot{W}_{out}$	W	Power output
DCr	km/kg	Distance covered per unit mass of fuel consumed
WGr	kJ/kg	Work generated per unit mass of fuel consumed

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1.1. Object

The purpose of this thesis is the study and understanding of a particular type of aircraft propulsion system, the turbofan, while cruising at different altitudes. The behaviour of this certain sort of airplane engine is predicted and analysed by the synergy of two different methods: at an early stage of the investigation, an interactive programmed model of a turbofan engine is used to simulate its performance within a realistic and delimited operating environment; later, based on the results obtained from the simulation, a more specific study of the core engine or gas generator (compressor, combustor and gas turbine) is carried out.

The tracking or study of the performance of a well-defined turbofan is brought out by obtaining the value of particular variables describing by its nature the behaviour of the aircraft engine. A couple of clarifying examples of that could be the engine's fuel consumption and the thrust or propelling force provided under its operation. These measures and its further correlation and manipulation enable a deeper study and understanding of the subject.

The software chosen to carry out the simulation is MATLAB Simulink, a data flow graphical programming language tool for modelling, simulating and analysing multidomain dynamic systems. This software offers a tool or block system, as referred to in the MATLAB's jargon, which facilitates the study of a turbofan engine by the procurement of two descriptive variables, both marketed as examples above. Starting out from a deep comprehension of the programmed system and its subsequent description, it is intended to relate all the inherent subsystems and blocks to the physical realities, the operational principles ruling the exercise of turbofans in the skies.

Moreover, and as a result of a complete understanding of the turbofan simulating system, as well as of the physical laws bounding it, the simulating system itself will be edited in order to get a desired set of results, which facilitate a deeper study of the core engine, or gas turbine system when operating under a specifically defined conditions: an airplane while cruising at different altitudes.

1.2. Scope

Today there are several different fanjet designs, and the number keeps rising. All the models used today, as well as those which are already obsolete, present smaller or bigger variances from a basic turbofan. This is, in few words, an engine operated by a gas turbine, which also drives a compressor and a fan.

The investigation driven in this thesis does not intend to focus on a particular type of turbofan but on a generic or basic one. However, at a certain point of the study, a set of specifications will be assumed in order to deepen into the comprehension of this type of aircraft propulsion system. Such specifications do inevitably narrow the investigation into a more defined field of study.

The simulation carried out in this paper is based on a programmed representation of a high-bypass turbofan and the precision of the recreated model is limited up to subsonic speeds of the aircraft. The reason why the program's outputting is no longer accurate beyond speeds higher than Mach number = 1 is because the simulator model is ruled by some equations, which are no longer precise for the engine's performance at values above this limit.

Besides these specifications, the simulating system does not operate under further constraining design parameters, but only within the boundaries set by the operational principles of a basic fanjet engine. On a first stage of the study, the simple nature of the simulating program delimits the research and facilitates the approach and handling to its block system. However, at a later phase it is necessary to contribute to the turbofan model design by defining other parameters so a more realistic portrait of the fanjet's performance is achieved.

1.3. Justification

Nowadays the tendency of passengers boarding in commercial flights is clearly growing year by year. And it is so, apparently no matter is the economic reality of the countries contributing the most to the mentioned increase of flight connections around the world.

To drive this huge amount of passengers all over the world, high capacity and long-range self-sustainable planes are used. And most of them are today operated by turbofan propulsion systems. Fanjets, as are also called, are a type of airbreathing jet engines comprising a gas turbine and a ducted fan that, using the mechanical energy from the turbine, accelerates air rearwards.

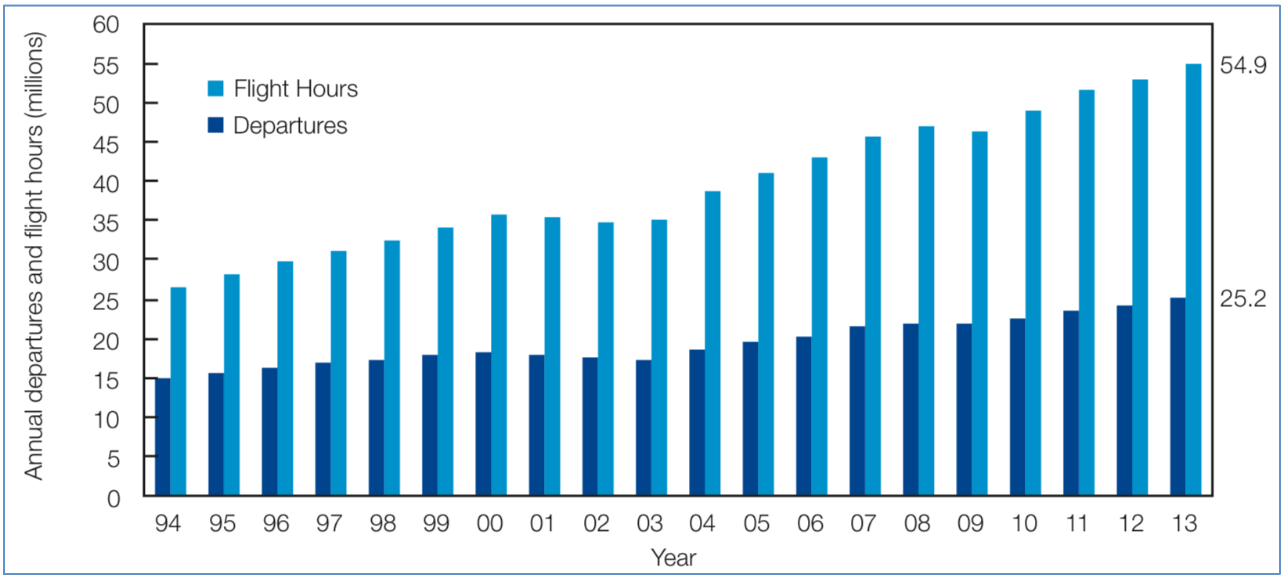

Figure 1: Annual departures and flight hours [Boeing Commercial Airplanes, 2014]

Turbofans used to drive commercial flights meet the designing parameters specified in the scope passage. Because of economical matters, efficiency of the aircraft system is primordial, thus commercial aircrafts' engines are of a high-bypass ratio type.

Their cruise velocities are between 700 km/h and 1000 km/h and are always operated within subsonic speeds, that is below Mach number 1.

As shown in the Figure 1, the number of flights operated has raised dramatically within the last twenty years, overtaking the figure of 25 million of departures for an annual period on 2013.

Through the study of the engine type driving the highest percentage of these flights, describing its operational principles and analysing its behaviour, it is intended to attain an understanding of how these figures are achieved and, if the range of operational altitudes among which airliners cruise is the most efficient from a purely technical point of view.

2. THEORETICAL FRAMEWORK

2.1. Aircraft propulsion systems

Propulsion is the act or process of pushing something forward. We humans are able to propel ourselves, this is to move, by transferring the energy processed by our body out from food, water and oxygen to our muscles, converting it into kinetic energy. Airplanes, in order to move, need a set of energy sources and a "metabolism" or system able to process all its inputs into kinetic energy, just as well as humans do. Obviously the comparison cannot be brought further, but the parallelism is evident.

Different propulsion systems develop thrust, the force which moves any aircraft through the air, in different ways, but all thrust is generated through some application of Newton's third law of motion; for every action there is an equal and opposite reaction.

All propulsion systems operate accelerating a working fluid that is driven through it, obtaining a reaction force opposite to that generated, that will propel the system. Out from the general thrust equation it is possible to appreciate that the amount of thrust generated depends on the mass flow passing through the engine and the exit velocity of the working fluid.

Assuming that the pressure of the working fluid is equal at the outlet as at the inlet of the engine, the force of thrust (F_T) generated is obtained by the following equation.

$$F_T = v_o \cdot \dot{m}_o - v_i \cdot \dot{m}_i$$

Equation 1

In a real operating fanjet, though, the pressure of the working fluid will not be equal at the outlet as at the inlet of the engine. Hence, the thrust equation is broaden by the addition of the pressure's difference times the area of the outlet.

$$F_T = v_o \cdot \dot{m}_o - v_i \cdot \dot{m}_i + (P_o - P_i) \cdot A_o$$

Equation 2

Airplane propulsion systems have evolved dramatically since the first flight, led by the Wright brothers on the early 20th century. The airship they designed was driven by propellers, a type of fan that transmits power by converting rotational motion into thrust. And so it was until the very end of the II World War, that every plane was driven by systems similar to that introduced by the American brothers, using propellers.

Both I and II World Wars supposed a huge investment on the innovation and development of aircrafts. The result was a dramatic expansion of the knowledge within this engineering field, widening solutions for common problems, which enlarged the amount of propulsion systems working under different operational principles.

During World War II a new type of airplane engine was developed independently in Germany and in England, in both cases for a military application. This engine was called a gas turbine engine, although it is also referred to as a jet engine. Aircraft propulsion systems driven by gas turbines are rotatory engines that extract energy from a flow of combusted gas. They have an upstream compressor coupled to a downstream turbine with a combustion chamber in-between. This set is not only used in aviation but in much more applications, such as power plants. When referring to aircraft engines, those three core components (compressor, turbine and combustion chamber) are often called the “gas generator”. There are many different variations of gas turbines, but they all use a gas generator system of some type.

2.2. Gas Turbines

Gas turbines have existed already for a long time. Although its design development did not have a broad application as for propelling aircrafts until the World War II, the first prototype is known to be proposed in the first century AD. The aeolipile, as it was called, is a bladeless radial steam turbine consisting on a vessel with two oppositely bent nozzles and a water container. As the container is heated up the steam is driven through two pipes up to the vessel, where the vapour is expelled through the nozzles creating a torque or moment of force.

Figure 2: Aeolipile illustration [Wikipedia.org]

However, this prototype did not have any operational application but as a toy, and it was not until around 1500s when the study of this type of engine was again resumed.
Today, many of the existing aircraft propulsion systems are driven by jet engines. Though, before discussing their operation principles and differences, it is important to know how a gas turbine operates and behaves.

2.2.1. Theory of operation

In an ideal gas turbine, gases undergo through three thermodynamic processes: an isentropic compression (entropy remains constant), an isobaric combustion (pressure remains constant) and an isentropic expansion. This state succession, which is repeated all over again, uninterruptedly as the incoming flow enters the turbine, is known as the Brayton cycle, named after George Brayton (1830-1892), the American engineer who developed it, although it was not proposed and patented until 1971 by the Englishman John Barber.
In a practical gas turbine though, some percentage of the mechanical energy is irreversibly transformed into heat when gases are compressed, no matter which kind of compressor is used, due to internal friction and turbulence. When passing through the combustion chamber, where heat is added driving to an increase of the gas' specific volume, there is a slight loss in pressure. During expansion amidst the stator and rotor blades of the turbine, irreversible energy transformation or ebbing occurs once again. So for a real life gas

turbine, as exposed by the last three statements, the compression will not be isentropic, the combustion will not be isobaric and, again, the expansion will not be isentropic, as at three stages energy ebbing occurs.

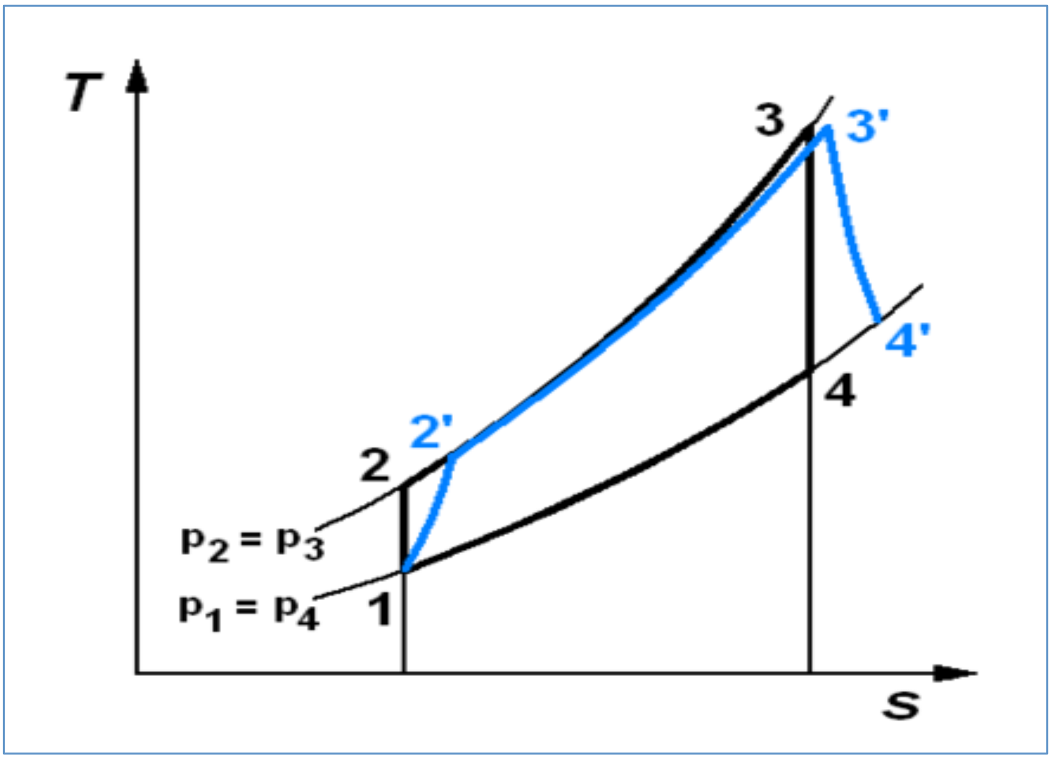

Figure 3: Idealized and real Brayton Cycle [Wikipedia.org]

As shown in the Figure 2: the idealized or theoretic Brayton cycle's diagram (in black) and the physically real one (in blue). This is a function of temperature (ordinate axis) and entropy (abscissa axis). The stage comprising the positions one and two (2' as well) correspond to the compression of the working fluid, isentropic in the ideal representation. From position 2/2' to 3/3' the fluid undergoes combustion and, finally, from 3/3' to 4/4' the fluid expands through the turbine, releasing energy. Although this state succession is marketed and referred to as a cycle, the gas escaping the turbine will not go through the process again. The air drawn into the compressor at state 1 from the surroundings is later returned to the surroundings at state 4 with a temperature greater than the ambient temperature. After interacting with the surroundings, each mass unit of discharged air would eventually return to the same state as the air entering the compressor, so the air passing through the components of the gas turbine are understood as undergoing a thermodynamic cycle. Hence, the diagram is closed as an understanding that the sequence is repeated while the incoming flow is uninterrupted, but not as a closed loop.

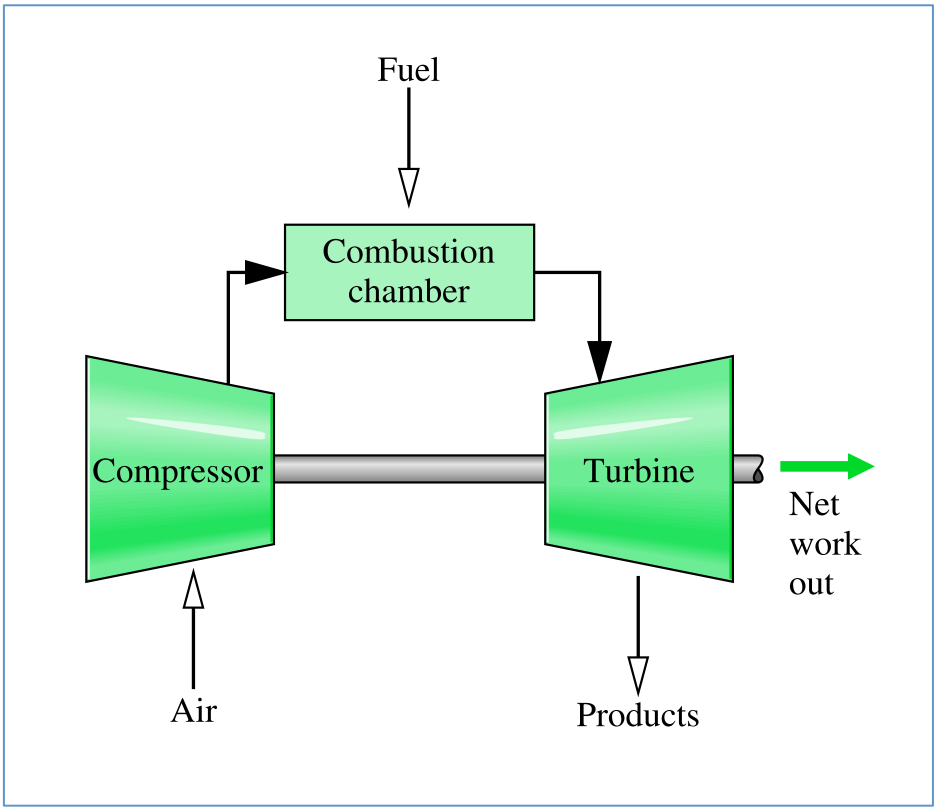

Figure 4: Simple gas turbine open to the atmosphere – [Moran & Shapiro, 2006]

2.2.2. Types of gas turbines

Gas turbines have many other applications besides propelling aircrafts. As the study driven in this thesis is focused in a particular and well-defined type of gas turbine, not all the known applications will be discussed. However, in order to broaden our understanding regarding their function, some of the most notorious and promising uses of gas turbine designs are shortly presented.

- **Power generation:** in electricity generating applications, the turbine is used to drive a synchronous generator, which provides the electrical power output. Usually, industrial gas turbines are of heavy construction as they are operated in a fixed station. However, the range of sizes vary largely, from man-portable mobile plants to enormous, complex systems weighing more than a hundred tonnes.

  Gas turbine power generators are used in different configurations depending on its operational purpose. In order to increase the efficiency, either a combined cycle or a cogeneration configuration might be applied. A combined cycle consists on recovering the hot exhaust gases from the gas turbine to power a conventional steam turbine, with both turbines being connected to electricity generators. This way, a greater part of the energy created within the combustion is profited.

On the other hand, the cogeneration or combined heat and power (CHP) system harness both heat and electrical power output generated along the cycle just as the combined cycle. The main difference remains in the direct application of the heat power of a cogeneration system to its particular purpose, with no need of an extra steam turbine.

These gas turbine power plants can be particularly efficient – up to 80% – when applying these cycles, way more than the gas turbine power plant used within aviation.

Microturbine: it is a type of combustion turbine on a relatively a small scale, producing both heat and electricity. Microturbines are becoming more attractive and used because of their compact size, lightweight and greater efficiency, as well as requiring relatively low capital and maintenance costs due to their size. Waste heat can also be used with these systems to achieve efficiencies greater than 80%. Depending on their application, they can output 25 kW to 500 kW and can be powered by natural gas, hydrogen, propane or diesel.

- **Propfan engine:** An unducted fan or ultra high-bypass turbofan, as it is also called, is a variation of a jet engine in which a gas turbine drives an unshielded propeller or fan. Propfans, conversely to turbofans, generate most of their thrust from the propeller and not from the exhaust jet. On the other hand, the primary difference between turboprop and propfan engine designs is that the propeller blades of an unducted fan are highly swept in order to allow them to operate at speeds around Mach 0.8, which is competitive with modern commercial turbofans. As follows, an illustration of a propfan throws some light at its operational conduct.

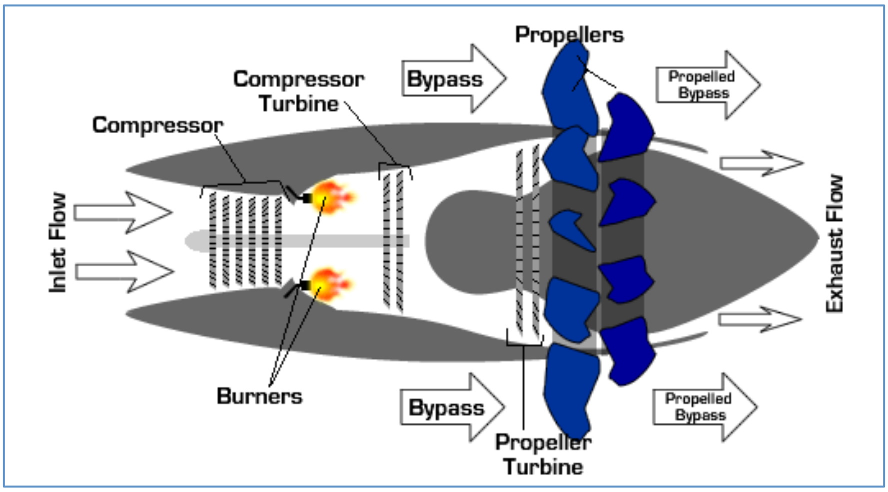

Figure 5: Propfan engine [smu.edu]

2.3. Turbofan engine

Nowadays, most of the aircraft propulsion systems in use are a variation of some type of a gas turbine engine. The turbojet, with or without the application of an afterburner, the turboprop, which have been shortly described, and the turbofan engines have most of their assembling parts in common, although the procurement of thrust is generated by slightly different operational principles.

The turbofan is, among all airship engines, the most used within civil aviation as well as in the military field. This is the main reason why the study will be focussed, from now on, on this type of aircraft propulsion systems.

A fanjet engine is the most modern variation of the basic gas turbine engine. As the other gas turbine engines, there is a core engine or gas generator, whose parts, as already mentioned, are a compressor, the burner or combustion chamber and the turbine. In this case, a ducted fan is situated at the front of the engine, contributing to the acceleration of the incoming airflow. At the rear of the turbofan, after the core engine, there is an additional turbine, which drives the fan. The fan and fan turbine, or additional turbine, are composed of many blades, like the core compressor and core turbine, and are connected by an additional shaft. Some of these blades, in the case of the fan as well as for the core compressor and turbines, remain stationary. The fan shaft, which might be called as low-pressure shaft, passes through the core shaft, or high-pressure shaft. This type of arrangement is so-called as a too-spool engine. Some engines have even more spools or shafts, although this will not be object of further study.

The fan, which is not present in turbojets, plays a key role in the generation of thrust by a fanjet. This is because fanjets divide the incoming airflow, bypassing a higher or lower percentage of it depending on its application. The stream of air entering the engine gains velocity as passes through the fan. At this point the flux is divided, either entering the compressor and undergoing the already described Brayton cycle through the gas generator, or being bypassed and directly expelled at the engine's exit. Thus, the turbofan produces thrust through the combination of these two portions, by-passed and non-bypassed air flux, working in concert; the so-called fan and jet thrust.

The ratio of mass-flow of air bypassing the engine core compared to the mass flow of air passing through the core is referred to as the bypass ratio. This is of capital interest for the industry as, depending on the field of operation of the aircraft, commercial or military, likewise the working requirements, as could be the necessity of bearing high loads in case of transportation of goods, the bypass ratio will be set to be one or another.

Engines that use more jet thrust relative to fan thrust are known as low bypass turbofans. Conversely those that operate generating considerably more fan thrust than jet thrust are known as high bypass. Most commercial aviation turbofan engines, because of their higher efficiency, are of the high-bypass type, while most modern military fighter engines are of low-bypass type.

Anyhow this particular specification will not receive further attention, as it does not meet the subject of investigation of the thesis.

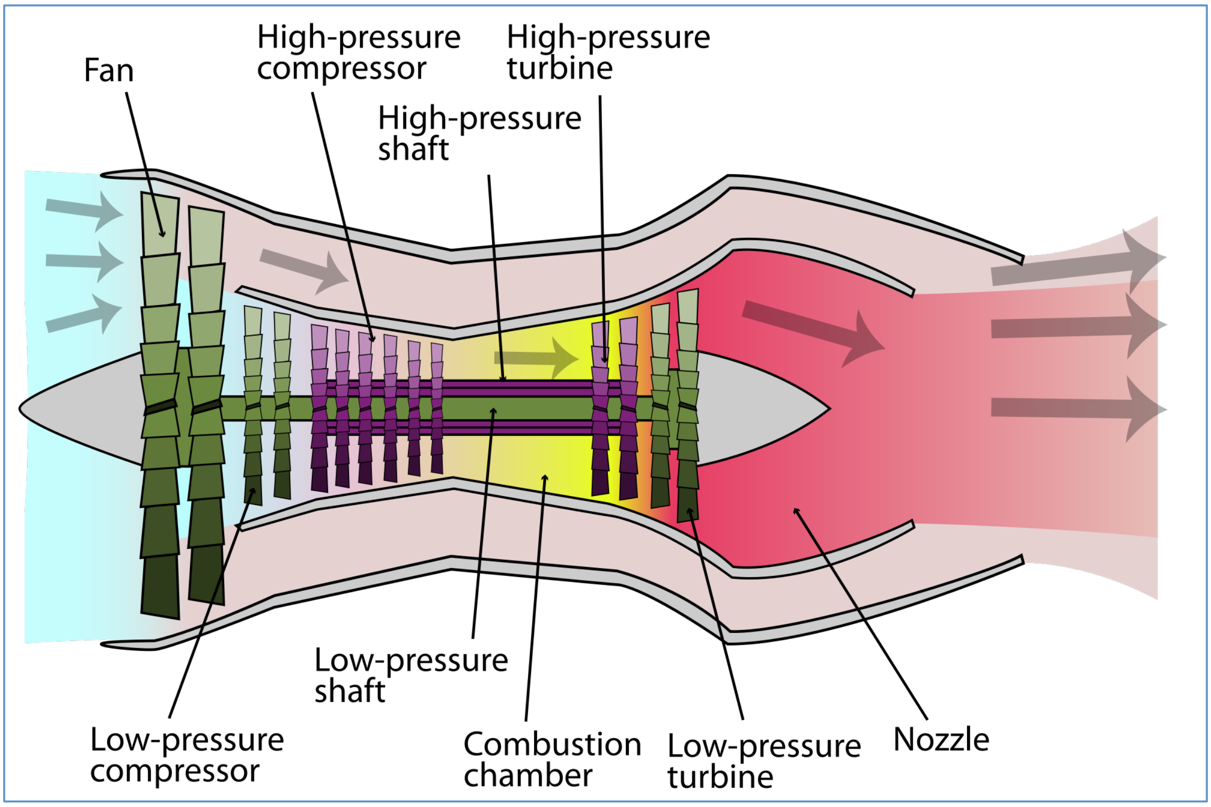

Figure 6: Schematic diagram of a high-bypass turbofan engine [Wikipedia.org]

2.4. Deciding the supporting software

In order to carry out the study of the turbofan and, more particularly of the core engine or gas turbine, a specific software is necessary to simulate the engine's behaviour and performance under different conditions. There are several software programs with which is possible to develop a study of this nature. Coming up next the programs that were considered for the study are presented to discuss which one fits the best the requirements set for the investigation and, hence, is the most appropriate.

2.4.1. Software portrayal

- **ANSYS CFD (Computational Fluid Dynamics)**: is a simulation software that allows to test and predict the impact of fluid flows in a virtual environment – throughout design and manufacturing as well as during end use. It enables the study of a wide range of different phenomena fluid flows: single or multi-phase, isothermal or reacting, compressible or not. Some clarifying examples of the studies which can be brought out using ANSYS CFD is the fluid dynamics of ship hulls, gas turbine engines (including the compressors, combustion chamber, turbine and afterburners), pumps, fans, etc.
- **MATLAB Simulink:** is a block diagram environment for multidomain simulation and model-based design. It supports simulation, automatic code generation and continuous test and verification of embedded systems.
  Simulink provides a graphical editor, customizable block libraries and solvers for modelling and simulating dynamic systems. It is integrated with MATLAB, enabling to incorporate MATLAB algorithms into models and export simulation results to MATLAB for further analysis.
  Capabilities:
    - Building the model: model hierarchical subsystems with predefines library blocks.
    - Simulating the model: simulate the dynamic behaviour of your system and view results as the simulation runs.
    - Analysing simulation results: view simulation results and debug the simulation.
    - Managing projects: easily manage files, components and large amounts of data for your project.
    - Connecting to hardware: connect your model to hardware for real-time testing and embedded system deployment.
- **EES (Engineering Equation Solver):** software package used for general equation-solving of coupled non-linear algebraic and differential equations. It provides many useful specialized functions and equations for the solution of thermodynamics and heat transfer problems, as well as providing uncertainty analysis, perform linear and non-linear regression and generating publication –quality plots, among many others features.
  It provides a high accuracy thermodynamic and transport property database containing hundreds of substances in a manner that allows it to be used with the equation solving capability.

2.4.2. Discussion

MATLAB Simulink offers within its block libraries a simulating system, comprised by a set of blocks, which are disposed in a certain manner to describe in two defining variables the behaviour of a turbofan engine. The range of inputs of this block system is limited to four, although it is possible to pre-set one of them. Besides this pre-programmed tool, MATLAB Simulink provides reference standards, environment models as well as aerodynamic coefficient importing options.

The software ANSYS CFD, which also provides the user of a wide range of tools, requires very large computational capacity whereas the MATLAB models are less demanding. Because of that, ANSYS simulating models are normally more time and memory consuming compared to that of MATLAB Simulink (Ramdenee, Ibrahim, Barka & Ilinca, 2013). ANSYS is considered to be the reference software for Finite Element Method models (FEM) (Fraisse, Ramousse, Sgorlon & Goupil, 2012), and FEM models are of capital importance within the research in the aviation industry.

The Engineering Equation Solver software provides the user of many implemented state functions as well as properties of a wide range of different gases. The generation of descriptive plots in two and three dimensions is easy to use and their display better than that of the software MATLAB Simulink. However, the tools provided to create a simulating environment, specifically for a turbofan engine, are not as broad as the ones offered by ANSYS or Mathworks.

Considering the time of investigation and previous knowledge of any of these software programs, the tools provided by MATLAB Simulink are of more interest, even when not using the finite element method, as they enable a broader study of the behaviour of a turbofan engine in a shorter time, because of its intuitive pre-programmed block systems.

2.5. Description of the MATLAB Simulink's Turbofan Engine System

The R2011a MATLAB Simulink version offers within the Aerospace Toolbox a system formed by a set of blocks, which disposition and programming perform the simulation of the behaviour of a turbofan engine. This block system computes the thrust and the mass fuel flow at a specific throttle position, Mach number and altitude, and given a set of assumptions and limitations.

The developer of the program and hence, of this block system, MathWorks, defines it using this words:

“This system is represented by a first-order system with unitless heuristic lookup tables for thrust, thrust specific fuel consumption (TSFC) and engine time constant. For the lookup table data, thrust is a function of throttle position and Mach number, TSFC is a function of thrust and Mach number and the engine time constant is a function of thrust. The unitless lookup table outputs are corrected for altitude using the relative pressure ratio δ and relative temperature ratio θ, and scaled by maximum sea level static thrust, fastest engine time constant at sea level static, sea level static thrust specific fuel consumption and ratio of installed thrust to uninstalled thrust.” (http://de.mathworks.com/help/aeroblks/turbofanenginesystem.html)

The figure 1 shows, as follows, the appearance of the simulator at a first glance.

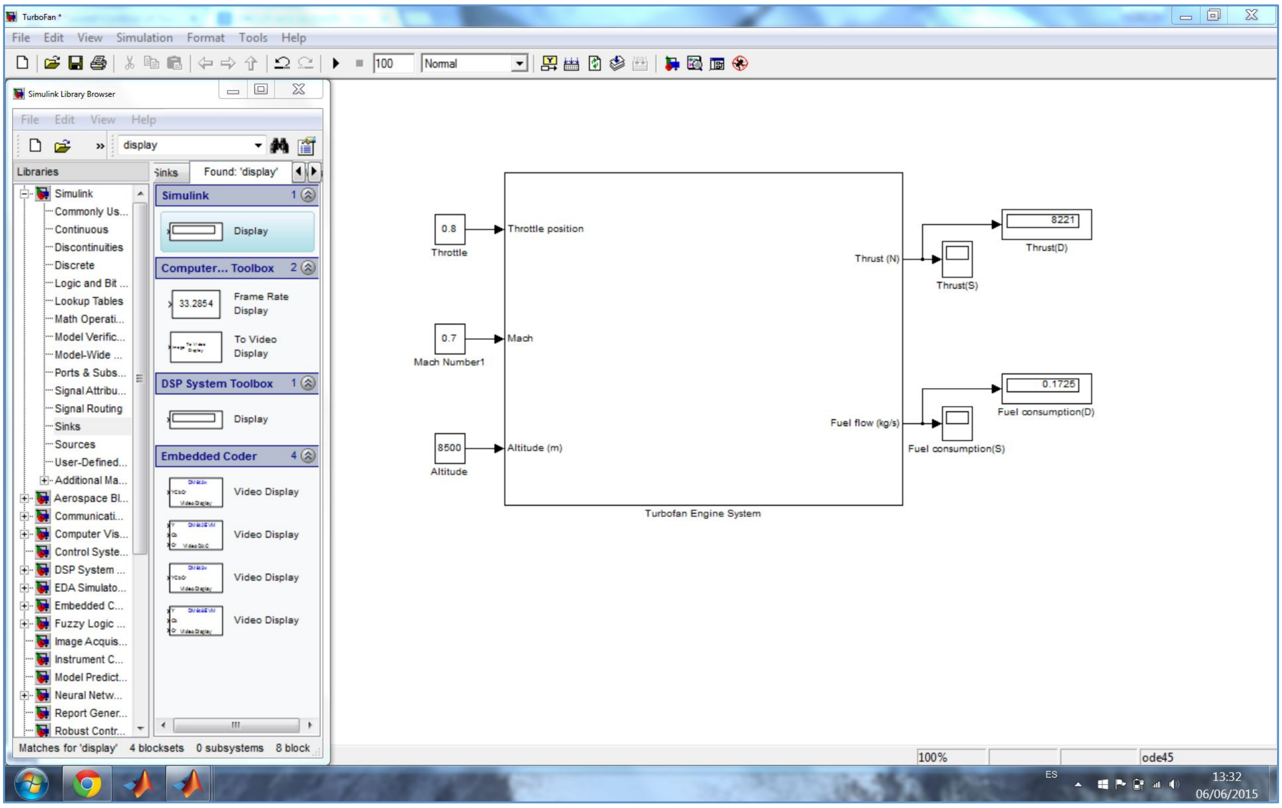

Figure 7: Main Turbofan Engine System block within the simulating interface [MATLAB Simulink]

2.5.1. Function Block Parameters

A set of specifications are disposed in the “Function Block Parameters: Turbofan Engine System” dialog window. It displays a brief and concise description of the block’s function and allows the definition of a group of parameters. The dialog box is showed in the figure below.

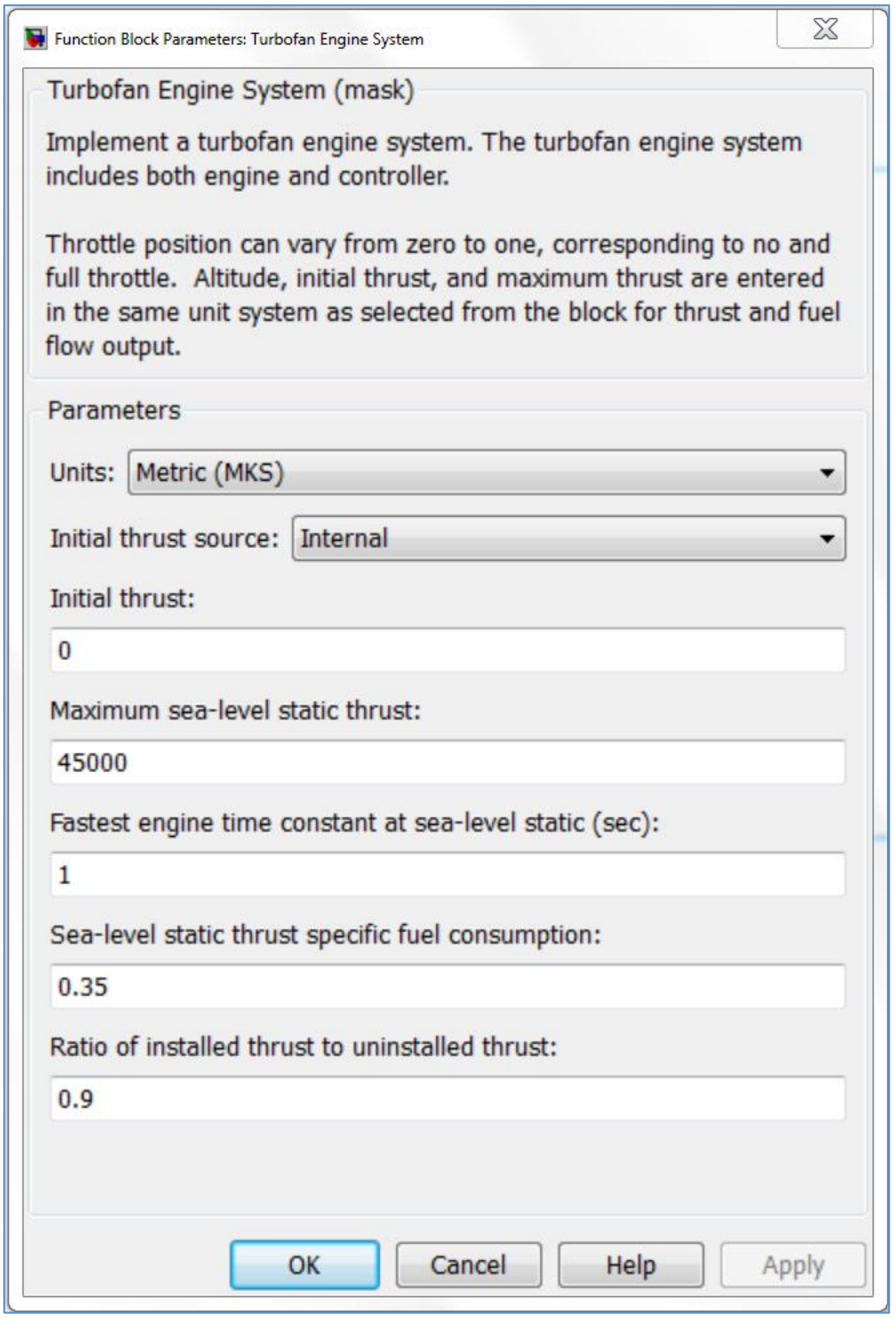

Figure 8: Turbofan Engine System block parameters [MATLAB Simulink]

As follows, the set of parameters which edition is enabled by the dialog window above presented are described:

- **Units:** it is possible to choose whether the inputs, operations and outputs' values are displayed and computed using Metric (MKS) or English units. The table 1 displays the basic dimensions present in the simulation, corresponding to each system of units.

	Metric	English
Length	Meter (m)	Feet (ft)
Mass	Kilogram (kg)	Pound (lb)
Time	Second (s)	Second (s)
Temperature	Kelvin (K)	Rankine (R)
Pressure	Pascal (Pa)	Pounds per square inch (psi)
Force	Newton (N)	Pound force (lbf)

Table 1: Metric and English units present in the block system [Author]

The metric unit system (MKS) is chosen to perform the present study. All the operations and default parameters are set in English units, although the block platform is programmed to allow the free selection of either one or the other unit system. When choosing the units corresponding to the International System of Units SI though, the outputs results are not precise. One of the parameters to be set in this dialog window, the 'Sea-level static thrust fuel consumption', has to be manually changed in order to obtain a set of outputs adequate to the SI system of units. The corresponding arrangement is discussed in its particular description.

- **Initial thrust source:** an option in the dialog box allows the user to choose if the amount of initial thrust desired to proceed with the simulation is inputted as an external source, along with the throttle position, the Mach number and the altitude, or if it is inputted in the Function Block Parameters dialog window. The simulation will be carried out with this parameter set internally.
- **Initial thrust:** amount of thrust generated by the system by the start of the simulation. Corresponds to the initial condition inputted in the integrator block (Figure 7). This parameter is set to be zero.
- **Maximum sea-level static thrust:** this is the highest amount of propelling force that the engine system will be able to generate at sea level and at Mach = 0. This is a value used as a reference of the engine's thrust output. The setting of this parameter is $F_{T,max}$ = 45000 N.
- **Fastest engine time constant at sea-level static (s):** it specifies the engine time lag or delay between the start of injection and the start of combustion at sea level and at Mach = 0.
- **Sea-level static thrust specific fuel consumption:** is the fuel consumption at sea level, at Mach = 0 and at maximum thrust. The TSFC or SFC describes the fuel efficiency of the engine with respect to thrust output, this is the mass of fuel needed to

provide a certain value of net thrust for a given time period: kilograms of fuel to second and kilonewton ($\frac{kg}{s \cdot kN}$). The setting of this parameter is, by default, 0,35 $\frac{lb}{lbf \cdot h}$.

- **Ratio of installed thrust to uninstalled thrust:** the uninstalled thrust is an ideal value derived from static tests on the engine, performed when this is still not assembled to the aircraft. Several operating constraints are neglected due to the nature of these tests, which do not dispose all the conditions met at a realistic working environment of an installed engine. The coefficient corresponding to this ratio represents the loss in thrust due to the mentioned installation. As the coefficient of this particular parameter will not be subject of further study, its value is not varied. The ratio of installed thrust to uninstalled thrust (IU_T) is set to be 0,9.

2.5.2. System's inputs and outputs

The inputs to the Turbofan Engine system are the following:

- **Throttle position:** throttle is the mechanism by which the working fluid flow is managed by constriction or obstruction. A butterfly valve is set to regulate the amount of working fluid entering the core engine. The actuation of this valve by the pilot or automatically by a computer, will increase or decrease the power generated by the propulsion system. The throttle position can vary from zero to one, corresponding to no and full throttle.
- **Mach number:** is a dimensionless quantity representing the ratio of a particle's velocity to the local speed of sound. This, the local speed of sound, and thereby the Mach number, depends on the condition of the surrounding medium, in particular the temperature and pressure. The equation describing the Mach number is the following:

$$M = \frac{v}{a}$$

Equation 3

- **Altitude:** as it is commonly known, is a distance measurement. In this particular case of study, within the aviation field, it is the distance of the operating system or fanjet from the mean sea level. In aviation the altitude is often referred to as geopotential height, which is not the same as geometrical height. The geopotential height is a vertical coordinate referenced to the Earth's mean sea level, which is adjusted by the variation of gravity acceleration depending on the latitude and elevation. This will be needed in a further stage of study. In any case, the value inputted as altitude is of geometrical height above the earth in meters.

The outputs of the Turbofan Engine System are the following:

- **Thrust:** is the propelling force generated by the fanjet under the set of specifications inputted. The unit of this output is Newton (N). The path to obtain its value will be described on a later stage of the study.
- **Fuel flow:** is the fuel consumed by the propelling system when generating thrust. The unit of this output is $\frac{kg}{s}$. The process followed to obtain its value will be described on a later stage of the study.

2.5.3. Subsystems

The above mentioned Turbofan Engine System is displayed at the Figure 1 at its simplest, only showing the set of inputs and outputs, which are primarily computed. Taking a look inside this block, or "under the mask" using the MATLAB's language, a deeper understanding of itself is possible. The next figure is a representation of the view obtained as "breaking" this outer shell.

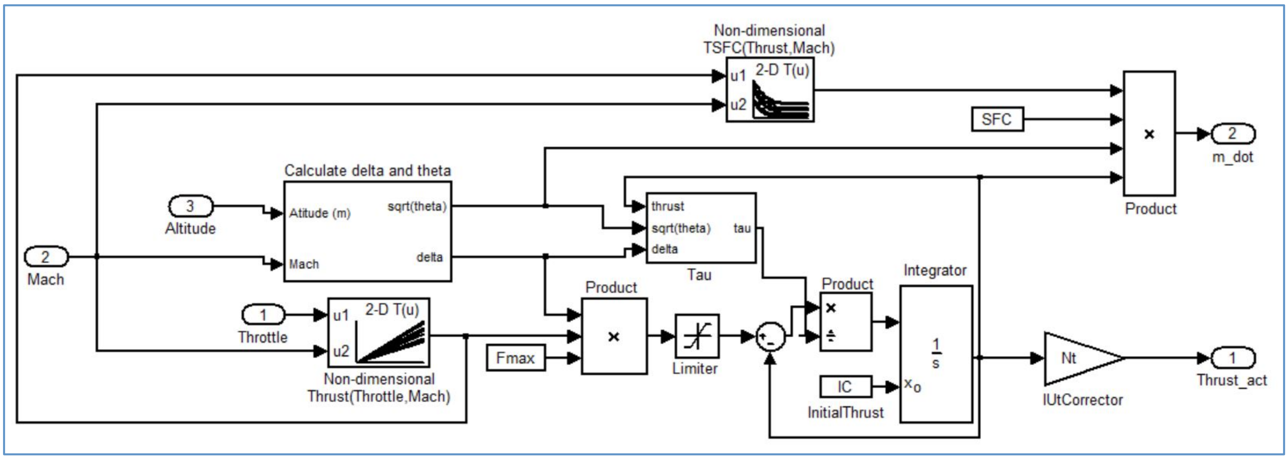

Figure 9: Main block subsystem [MATLAB Simulink]

In order to comprehend the meaning and function of each block taking part on the system presented above, all of them will be described and related to the operational principles of a turbofan engine.

- **Calculate delta and theta:**

Theta (θ) and delta (δ) are the relative temperature ratio and relative pressure ratio respectively. The relative temperature ratio θ is the relation, set as the fraction, of the stagnation temperature under the conditions of the operating environment, to a reference temperature, relative to sea level standard atmospheric conditions. This is:

$$\theta = \frac{T_{stagnation}}{T_{ref}}$$

Equation 4

The relative pressure ratio is the stagnation or total pressure of the air to the temperature at a chosen reference station relative to sea level standard atmospheric conditions. This is:

$$\delta = \frac{P_{stagnation}}{P_{ref}}$$

Equation 5

Although the default block system does not output the relative density ratio σ, its value is obtained along the process. The ratio relates the stagnation density to the density at a chosen reference station, relative to sea level standard atmospheric conditions. This is:

$$\sigma = \frac{\rho_{stagnation}}{\rho_{ref}}$$

Equation 6

The ratios described by the above mentioned equations are needed on later stages of the system and thus further discussed. Here is how to approach their values.

Taking a look underneath this particular block, even to a deeper level, it is possible to achieve a better understanding of its performance.

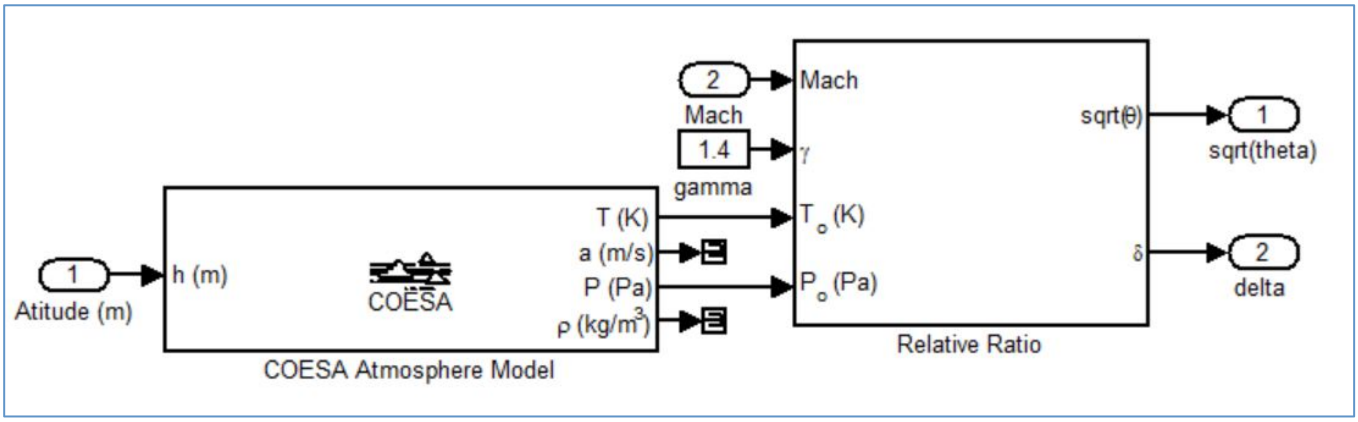

Figure 10: Calculate delta and theta block subsystem [MATLAB Simulink]

This subsystem is ran by the inputting of three characters: the altitude of operation, the Mach number or relative velocity of the system (aircraft) and the heat capacity ratio or adiabatic index γ.

The adiabatic index γ is the ratio of heat capacity or amount of heat needed to raise the temperature of an object a certain degree, at constant pressure (c_P) to heat capacity at constant volume (c_V).

$$\gamma = \frac{c_P}{c_V}$$

Equation 7

As the air is assumed to be a calorically perfect gas γ has a constant value of approximately 1,4 among the range of atmospheric temperatures under which the study is performed (-100ºC – 100ºC).

The COESA (Committee on Extension to the Standard Atmosphere) Atmosphere Model is a U.S. Standard Atmosphere model that defines values for atmospheric temperature, density, pressure, acceleration caused by gravity, sound speed, thermal conductivity as well as many other properties over a wide range of altitudes. Based on rocket and satellite data and perfect gas theory, the atmospheric densities and temperatures are represented from sea level to 1000 km. However, the block implemented by MATLAB Simulink limits the existing data up to 84852 m above the mean sea level. In any case, commercial aircrafts cruise somewhere between 9 km and 15 km above mean sea level, so the range of altitudes provided by the MATLAB's COESA Atmosphere model is wide enough.

This block is a subroutine or subsystem that accepts altitude as an input argument and returns values of temperature, pressure and density.

Among the different versions available, the 1976 U.S. Standard Atmosphere is the most recent one and the one to be used.
A quick view to a part of the data used in this model is showed in the table 1.

Geopotential altitude above MSL (m)	Static pressure (Pa)	Standard temperature (K)	Temperature Lapse Rate (K/m)
0	101325	288.15	-0.0065
11000	22632.1	216.65	0.0
20000	5474.89	216.65	0.001
32000	868.019	228.65	0.0028

Table 2: U.S. Standard Atmosphere Model, 1976 [ntrs.nasa.gov]

The COESA Atmosphere model block outputs four values: the local static temperature, the local speed of sound, the local pressure and finally the local density of the air. Both the local speed of sound and the local static density of the air are set by default to be terminated, as their values are of any interest to compute the system's outputs. On the other hand, the local static temperature and pressure of the air are inputted along with the Mach number and the heat capacity ratio γ on the Relative Ratio block. This one is in charge of computing the relative temperature ratio θ and relative pressure ratio δ. In order to understand the process in doing so, it is necessary to take a look under the mask of the block.

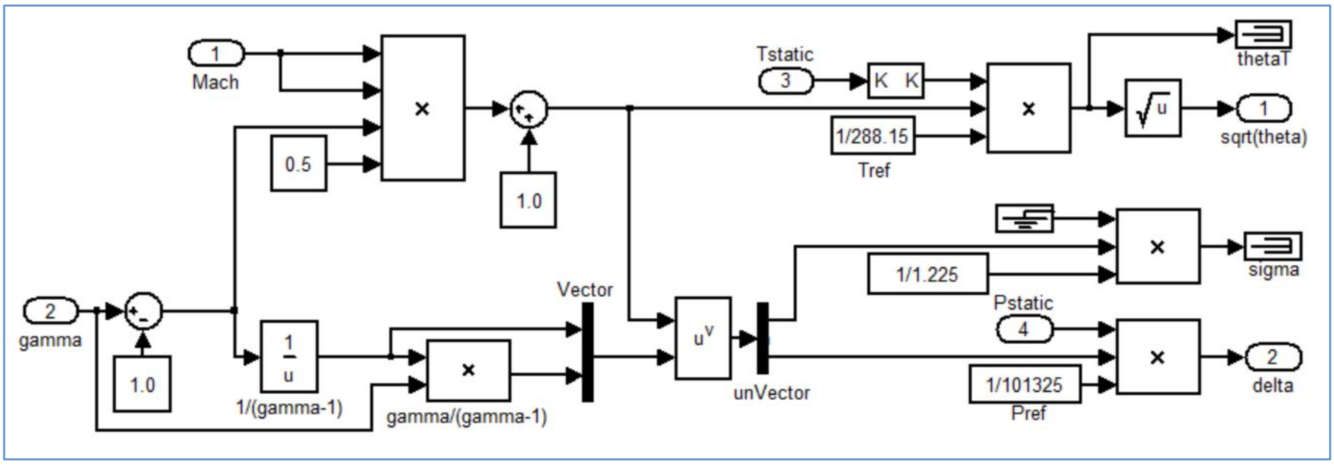

Figure 11: Relative Ratio block subsystem [MATLAB Simulink]

Both the temperature and pressure inputted along with the Mach number and the heat capacity ratio γ, are static or ambient values of the air. But, in order to compute delta and

theta, or the relative pressure and temperature ratios, it is necessary to calculate the temperature and pressure at the stagnation point. The stagnation point is a position in a flow field where the local velocity of the fluid is zero. These points exist at the surface of objects situated within a flow field, where the fluid is brought to rest respect the object. The temperature, as well as the pressure and density of the air are measured during operation of an aircraft by a probe mounted on its surface. This probe is designed to bring the air to rest relative to the airplane. As that happens, the kinetic energy of the air is converted into internal energy. The air is compressed and experiences an adiabatic increase in temperature. Therefore the values of temperature, pressure and density of the air obtained in flight are not accurate to the medium surroundings reality. To obtain a set of values closer to reality, the stagnation temperature, pressure and density of the air are related to the standard values provided by the COESA Atmosphere model.
The relation between the stagnation temperature and the static (or ambient) temperature is presented as follows:

$$T_{stagnation} = T_{static} \cdot (\frac{M^2 \cdot (\gamma - 1)}{2} + 1)$$

Equation 8

The relation between the stagnation pressure and the static pressure is presented as follows:

$$P_{stagnation} = P_{static} \cdot (\frac{M^2 \cdot (\gamma - 1)}{2} + 1)^{(\frac{1}{\gamma - 1})}$$

Equation 9

The relation between the stagnation density and the static density is presented as follows:

$$\rho_{stagnation} = \rho_{static} \cdot (\frac{M^2 \cdot (\gamma - 1)}{2} + 1)^{(\frac{\gamma}{\gamma - 1})}$$

Equation 10

The three relations above presented are obtained out from the isentropic flow equation development for a calorically perfect gas. Isentropic flows occur when a fluid stream approaches the speed of sound and, because of compressibility effects, the density of the fluid varies from one location to next, contributing as also noted to the variation of the air pressure and temperature.

The reference values relative to sea level standard atmospheric conditions used in obtaining theta θ, delta δ and sigma σ are presented in the next table.

T_{ref}	P_{ref}	ρ_{ref}
15 ºC	1 atm	$1{,}225 \cdot 10^{-3}$ g/cm^3
288,15 K	101325 Pa	1,225 kg/m^3

Table 3: Reference values relative to sea-level standard atmospheric conditions [Author]

- **Non-dimensional thrust:**

Taking a step back again to a brought view of the complete system (Figure 2) and in a parallel stage to the Delta and Theta's block, the amount of thrust or propelling force created is approached from the inputs of the Mach number and Throttle position, which are indexed in a two-dimension interpolated table. The table is a sampled representation of a function in two variables. Input sets are related to positions in the table, each one corresponding to a particular output. Depending on the inputs' values, a particular output is computed. As both Mach number and throttle position lack of dimension or units, the output is as well a non-dimensional value, a ratio of the thrust generated to the maximum possible engine's output.

The look-up table performed by this block is presented as follows.

	Mach number										
	0	0.1	0.2	0.3*	0.4	0.5	0.6	0.7	0.8	0.9	1
Throttle position	**Non-dimensional Thrust**										
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0.92	0.84	0.76	0.68	0.6	0.58	0.56	0.54	0.52	0.5

Table 4: Non-dimensional Thrust(Throttle, Mach) look-up table [MATLAB Simulink]

*The exact value indexed is: 0.30000000000000004

To obtain the non-dimensional thrust corresponding to a throttle input different to zero or one, the block performs an interpolation between the indexed values and the desired throttle value. The linear interpolation formula is described in the equation 12.

$$y = y_a + (y_b - y_a) \cdot \frac{(x - x_a)}{(x_b - x_a)}$$

Equation 11

Where (x_a, y_a) and (x_b, y_b) are the known data points closest and limiting above and below the vale corresponding to the data point to obtain.

Thus, the non-dimensional thrust output corresponding to a Mach number = 1 and a throttle position = 0,7 will be computed in the following manner:

$$y = 0 + (0.5 - 0) \cdot \frac{(0.7 - 0)}{(1 - 0)} = 0.35$$

Example 1

Considering (x_a, y_a) = (0, 0); (x_b, y_b) = (1, 0.5) and (x, y) = (0.7, y), where x's stand for the throttle position and y's for non-dimensional thrust.

- **Non-dimensional Thrust Specific Fuel Consumption:**

This block, as well as the one above described, is a two-dimensional look-up table. The inputs that enable the indexation and further outputting of the corresponding value are the Mach number, again, and the output received from the Non-dimensional Thrust block. In this case none of the inputs have dimensions or units either, so the output will be a dimensionless character as well. This particular interpolated table is presented as follows.

	Mach number										
	0	0.1	0.2	0.3*	0.4	0.5	0.6	0.7	0.8	0.9	1
Non-dimensional thrust	**Non-dimensional TSFC**										
0	2.06	2.16	2.31	2.83	2.97	3.11	3.35	3.59	3.62	3.66	3.7
0.04	1.8437	1.9437	2.0937	2.5565	2.6965	2.8365	3.0597	3.2997	3.3297	3.1192	3.1592
0.08	1.6618	1.7618	1.9118	2.3228	2.4628	2.6028	2.8092	3.0492	3.0792	3.1192	3.1592
0.12	1.5104	1.6104	1.7604	2.1248	2.2648	2.4048	2.5948	2.8348	2.8648	2.9048	2.9448
0.16	1.3861	1.4861	1.6361	1.9587	2.0987	2.2387	2.413	2.653	2.683	2.723	2.763

	Mach number										
	0	0.1	0.2	0.3*	0.4	0.5	0.6	0.7	0.8	0.9	1
Non-dimensional thrust	**Non-dimensional TSFC**										
0.20	1.2854	1.3854	1.5354	1.8206	1.9606	2.1006	2.2603	2.5003	2.5303	2.5703	2.6103
0.24	1.2052	1.3052	1.4552	1.7074	1.8474	1.9874	2.1335	2.3735	2.4035	2.4435	2.4835
0.28	1.1425	1.2425	1.3925	1.6156	1.7556	1.8956	2.0295	2.2695	2.2995	2.3395	2.3795
0.32	1.0948	1.1948	1.3448	1.5425	1.6825	1.8225	1.9455	2.1855	2.2155	2.2555	2.2955
0.36	1.0594	1.1594	1.3094	1.4851	1.6251	1.7651	1.8789	2.1189	2.1489	2.1889	2.2289
0.40	1.0342	1.1342	1.2842	1.441	1.581	1.721	1.8271	2.0671	2.0971	2.1371	2.1771
0.44	1.017	1.117	1.267	1.4079	1.5479	1.6879	1.7879	2.0279	2.0579	2.0979	2.1379
0.48	1.0061	1.1061	1.2561	1.3837	1.5237	1.6637	1.7592	1.9992	2.0292	2.0692	2.1092
0.52	0.9999	1.0999	1.2499	1.3666	1.5066	1.6466	1.739	1.979	2.009	2.049	2.089
0.56	0.9969	1.0969	1.2469	1.3548	1.4948	1.6348	1.7256	1.9656	1.9956	2.0356	2.0756
0.60	0.9959	1.0959	1.2459	1.347	1.487	1.627	1.7176	1.9576	1.9876	2.0276	2.0676
0.64	0.9961	1.0961	1.2461	1.3421	1.4821	1.6221	1.7134	1.9534	1.9834	2.0234	2.0634
0.68*$_1$	0.9968	1.0968	1.2468	1.339	1.479	1.619	1.712	1.952	1.982	2.022	2.062
0.72	0.9972	1.0972	1.2472	1.337	1.477	1.617	1.7123	1.9523	1.9823	2.0223	2.0623
0.76	0.9973	1.0973	1.2473	1.3357	1.4757	1.6157	1.7136	1.9536	1.9836	2.0236	2.0636
0.80	0.9969	1.0969	1.2469	1.3346	1.4746	1.6146	1.7152	1.9552	1.9852	2.0252	2.0652
0.84	0.9961	1.0961	1.2461	1.3339	1.4739	1.6139	1.7168	1.9568	1.9868	2.0268	2.0668
0.88	0.9954	1.0954	1.2454	1.335	1.4735	1.6135	1.718	1.958	1.988	2.028	2.068
0.92	0.9952	1.0952	1.2452	1.334	1.474	1.614	1.7188	1.9588	1.9888	2.0288	2.0688
0.96	0.9964	1.0964	1.2464	1.3359	1.4759	1.6159	1.7194	1.9594	1.9894	2.0294	2.0694
1	1	1.1	1.25	1.34	1.48	1.62	1.72	1.96	1.99	2.03	2.07

Table 5: Non-dimmensional TSFC(Thrust, Mach) look-up table [MATLAB Simulink]

*$_1$ The exact value indexed is 0.6799999999999999

- **Tau:**

The so-called Tau block computes a variable by means of a subsystem consisting of three inputs: gross thrust, delta (δ) and the square root of theta ($\sqrt{\theta}$). This subsystem is displayed in the figure 10.

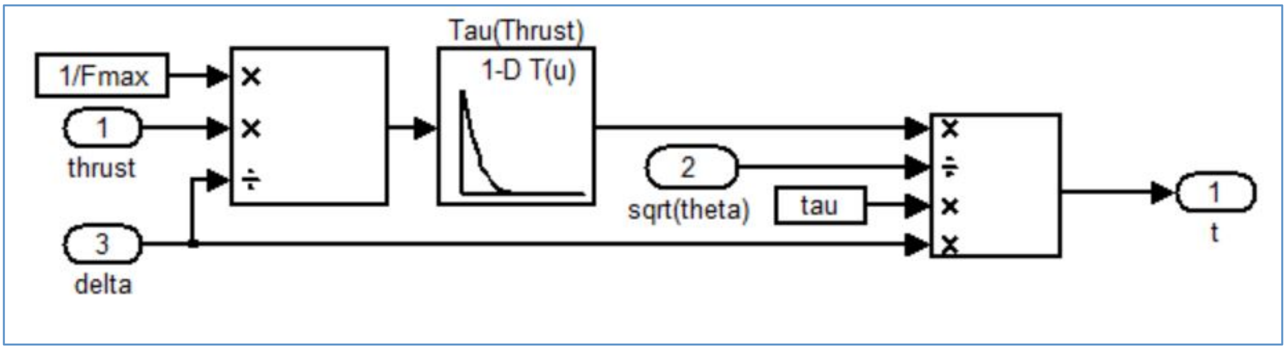

Figure 12: Tau block subsystem [MATLAB Simulink]

At a first stage, the gross thrust and δ are inputted to a logical operator block where, along with a set constant, which is the inverse of the maximum sea-level static thrust, are operated in the manner described in the equation presented below.

$$\frac{1 \cdot F_T}{\delta \cdot F_{T,max}}$$

Equation 12

Where F_T stands for the gross thrust.

The result of the operation described by the Equation 13 is a coefficient of the thrust output relative to the reference of maximum thrust at sea-level ($F_{T,max}$) and to the conditions at which the engine is operating (δ). This value will be from now on referred to as 'thrust coefficient at operating conditions'.

The value computed out by this block, the thrust coefficient at operating conditions, is indexed to a one-dimension look-up table, which according to the character introduced, selects its corresponding output. The value outputted is another coefficient: the fastest engine time constant at sea level static. This parameter, primarily set in the block system's dialog box, is now ranged within the operating conditions. From now one it will be referred to as 'Tau coefficient'.

The mentioned look-up table is displayed as follows:

Thrust coefficient at operating conditions	Tau coefficient
0	5,18
0,04	4,14
0,08	3,32
0,12	2,68
0,16	2,19
0,2	1,83
0,24	1,56
0,28	1,33
0,32	1,23
0,36	1,15
0,4	1,1
0,44	1,06
0,48	1,04
0,52	1,03
0,56	1,02
0,6	1,01
0,64	1
0,68	1
0,72	1
0,76	1
0,8	1
0,84	1
0,88	1
0,92	1
0,96	1
1	1

Table 6: Tau(Thrust) look-up table [Author]

As it can be noted, for a higher 'thrust coefficient at operating conditions' the 'tau coefficient' is smaller up to one. This is because, at full thrust, the engine time lag is lower than at low revolutions.

The character obtained out from the Tau(Thrust) look-up table is operated, along with the square root of theta ($\sqrt{\theta}$), delta (δ) and a second constant (s = 1), which corresponds to the 'fastest engine time constant at sea-level static' parameter, by a logical operator block in the manner described as follows.

$$\tau = \frac{T(thrust) \cdot s \cdot \delta}{\sqrt{\theta}}$$

Equation 13

Where *T(thrust)* stands for the value outputted by the previous look-up table and *tau* is the fastest engine time constant at sea-level static.

The result obtained out of this subsystem is τ, the engine time constant at the conditions under which the engine operates.

2.5.4. Obtaining the thrust output

The propelling force generated by the engine system is procured by operating several outputs of various subsystems. Here are described the operations implemented up to obtaining the thrust output.

The value obtained out from the 'Non-dimensional thrust' look-up table is multiplied along with the relative pressure ratio (δ) and the maximum sea-level static thrust ($F_{T,max}$) (Figure 7). The result of this product, however, is limited downwards up to the value of the maximum sea-level static thrust $F_{T,max}$ by a limiter block, as this is largest amount of thrust that the fanjet can generate.

$$F_{T,max} \cdot T(ndt) \cdot \delta$$

Equation 14

The resulting character of the product shown above is the amount of thrust generated by the propulsion system, corresponding to the input controls. The output of this, or any particular value of propelling force, however, is not instantaneous. Thus, after being limited up to the maximum pre-set value, the thrust character is corrected by a feedback, adjusting its value along the simulation as a time function.

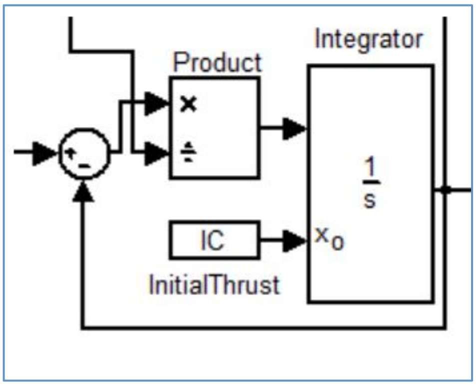

Figure 13: Closer look into the thrust corrector feedback [MATLAB Simulink]

This close loop enables the integration of the whole range of values outputted as thrust along the simulation, which may vary from the ignition time or first simulation step to the final outputting of the propelling force corresponding to the input setting controls. This is done taking into account the engine time lag τ.

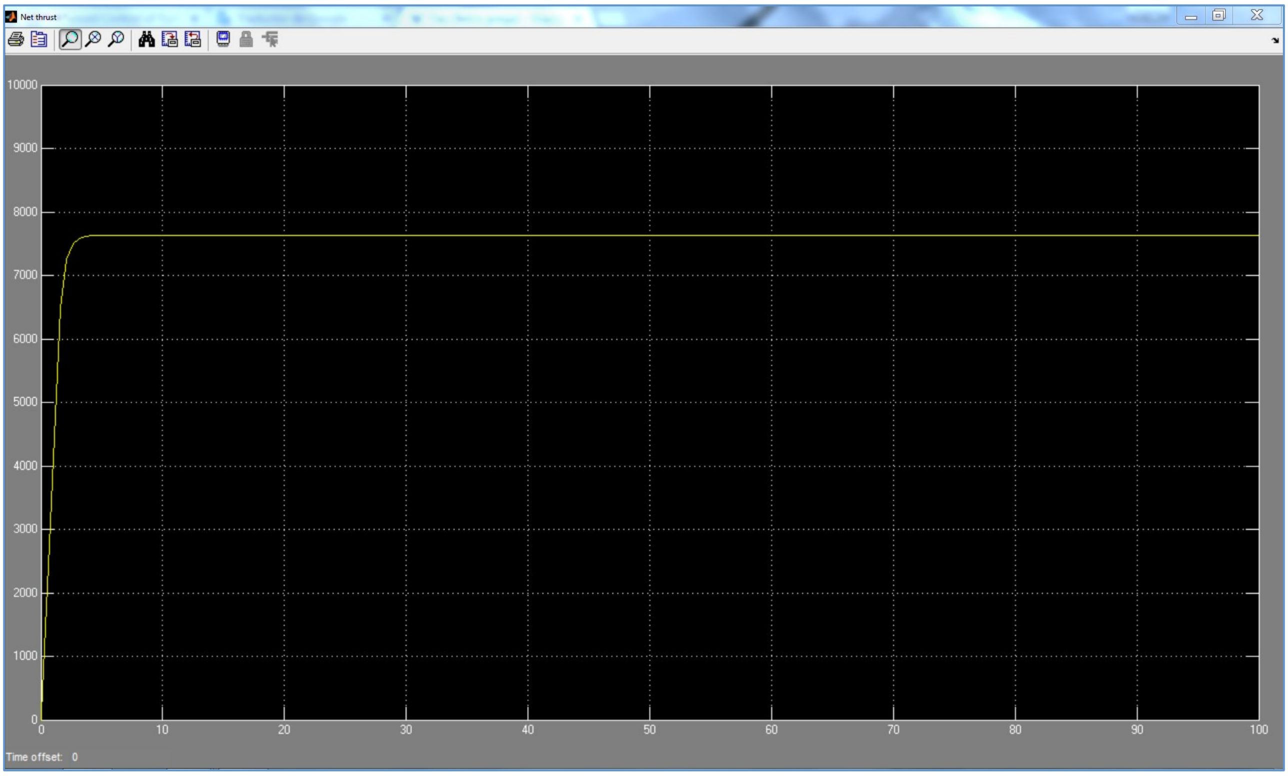

Figure 14: Thrust output graph; Newton (ordinate axis), seconds (abscissa axis) [MATLAB Simulink]

Finally, the gross thrust ($F_{T,gross}$) is computed by correcting, again, the value of gross thrust:

$$F_T \cdot IU_T$$

Equation 15

Where IU_T is the ratio of 'installed thrust to uninstalled thrust' parameter set at the Turbofan Engine System's functions dialog window.

2.5.5. Obtaining the fuel consumption

The fuel consumption value is obtained by operating several outputs of various subsystems. The integrated value of gross thrust, which procurement has been detailed previously, along with the square root of theta ($\sqrt{\theta}$), the 'sea-level static thrust specific fuel consumption' coefficient, parameter set in the Turbofan Engine System dialog box, and the output of the 'Non-dimensional thrust specific fuel consumption' look-up table are multiplied.

$$T(tsfc) \cdot SFC \cdot F_{T,net} \cdot \sqrt{\theta}$$

Equation 16

The character obtained out from this operation is the fuel flow in $\frac{kg}{s}$.

2.6. Edition of the MATLAB Simulink's Turbofan Engine System

To perform the intended study some adjustments to the simulating system provided by MATLAB Simulink must be done. Coming up next, the reasons to edit the simulator are specified and its development described.

- Unit System:

  The unit system used all along the study as well as for the simulation is the Metric or International System of Units. As it has been presented, the software enables the user to choose among the metric (MKS) and English systems of units, while it is set by default to use the last quoted. The system should rearrange itself depending on the choice made. However, there is a character of key importance, the 'sea-level static thrust specific fuel consumption', that is not automatically varied whatever the unit system chosen is. Therefore it has been done manually. Converted to the metric unit system and later to a more manageable set of units:

$$0{,}35\ \left[\frac{lb}{lbf \cdot h}\right] \cdot \frac{0{,}45359237\ [kg]}{1\ lb} \cdot \frac{1\ lbf}{4{,}44822162\ [N]} \cdot \frac{1h}{3600[s]} = 9{,}914 \cdot 10^{-6} \left[\frac{kg}{N \cdot s}\right] =$$

$$= 9{,}914 \cdot 10^{-3} [\frac{kg}{kN \cdot s}] = 9{,}914\ [\frac{g}{kN \cdot s}]$$

Equation 17

In the Turbofan Engine System dialog window, the 'sea-level static thrust specific fuel consumption' is changed to its value corresponding to $\frac{kg}{N \cdot s}$.

- Outputs added:

  There is a group of variables computed by the system, which are either terminated or simply not displayed within the final set of outputs. All the characters computed by the COESA Atmosphere model block are essential to perform the study of the gas generator, so they have been brought to a first sight as principal outputs.

  Besides the values of the stagnation temperature and pressure are necessary to approach the air mass flow when considering the effects of compressibility, natural at velocities close to the speed of sound.

  All together, the simulating software predicts and outputs the values of the following characters: thrust force and fuel consumption (default); atmospheric pressure and temperature, speed of sound, density of the air and stagnation temperature and pressure.

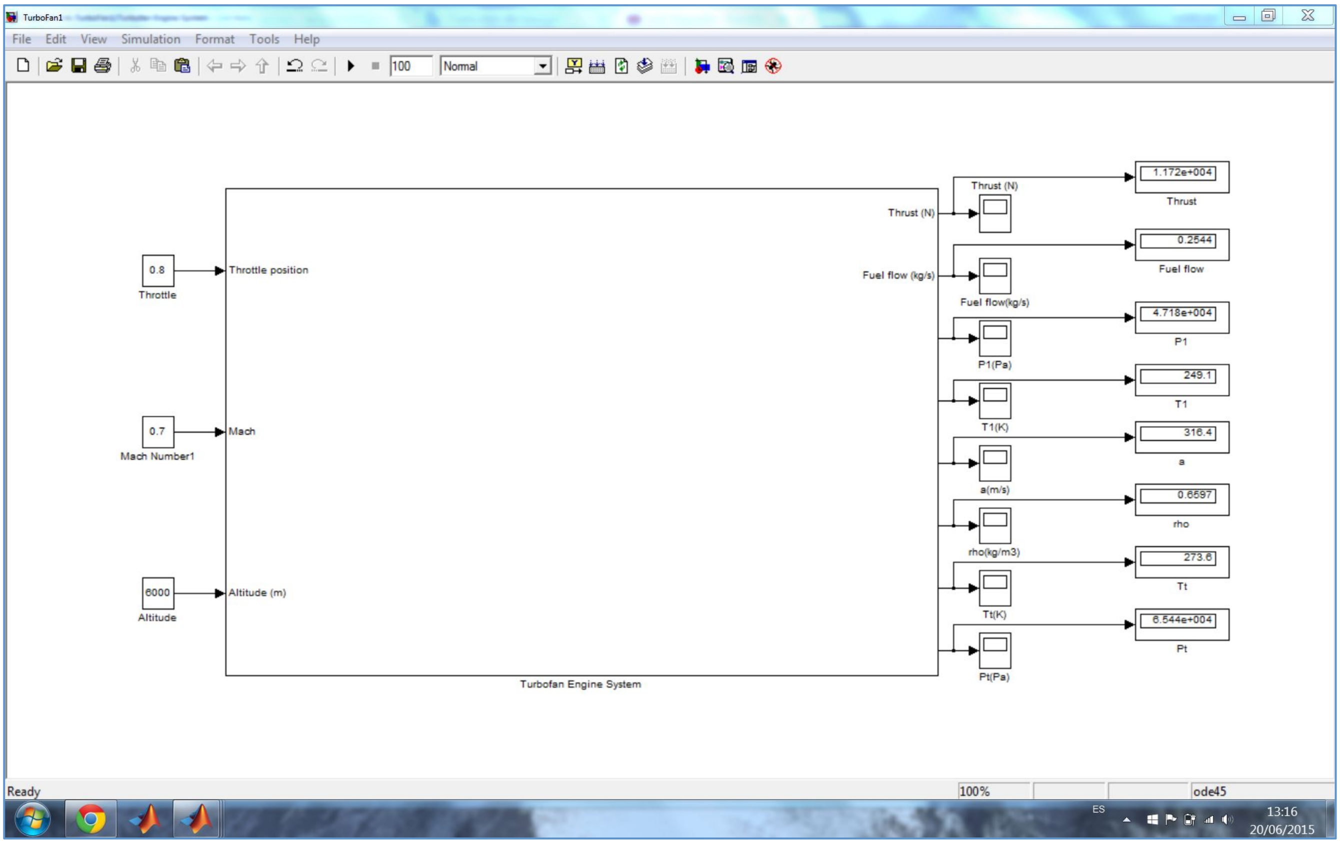

Figure 15: Main Turbofan Engine System block edited [MATLAB Simulink]

3. EXPERIMENTAL FRAMEWORK

3.1. Design and operational assumptions

3.1.1. Design assumptions:

The turbofan block system provided by the software MATLAB Simulink does not define any of the engine's components, but computes the net thrust produced and the fuel consumed by setting a limited group of describing values, which have been already specified and portrayed. In order to obtain further information of the turbofan's performance, which enables the study to deepen, it is necessary to define it with higher precision, delimiting its behaviour with new designing parameters: the fan diameter, the overall pressure ratio of the engine, and its bypass-ratio.
To set some references, some state of the art high-bypass commercial fanjets' parameters are tabulated as follows:

Name	Trent 900	GE9X
Developer	Rolls-Royce	General Electrics
Fan Diameter	116" (2,95 m)	128" (3,25 m) - 132" (3,35 m)
Compressor Pressure Ratio	39:1	60:1
Max. Thrust at Sea Level	311375 N	442598 N
Bypass ratio	8,5-8,7:1	10,3:1

Table 7: Trent 900 & GE9X turbofan engines characteristics [geaviation.com, rolls-royce.com]

Both the fan diameter and the compressor pressure ratio of a commercial high-bypass turbofan vary broadly depending on their performance specifications. The GE9X is one of the newest fanjet engines in the market, and its overall pressure ratio is the highest among all the other aircraft propulsion systems up to date. On the other hand, this makes the engine heavier, as the compressor needs more stages to achieve such pressure characters. However, the study to determine these two designing parameters does not meet the object of investigation of the present paper. Thus, the values describing these parameters will be assumed.

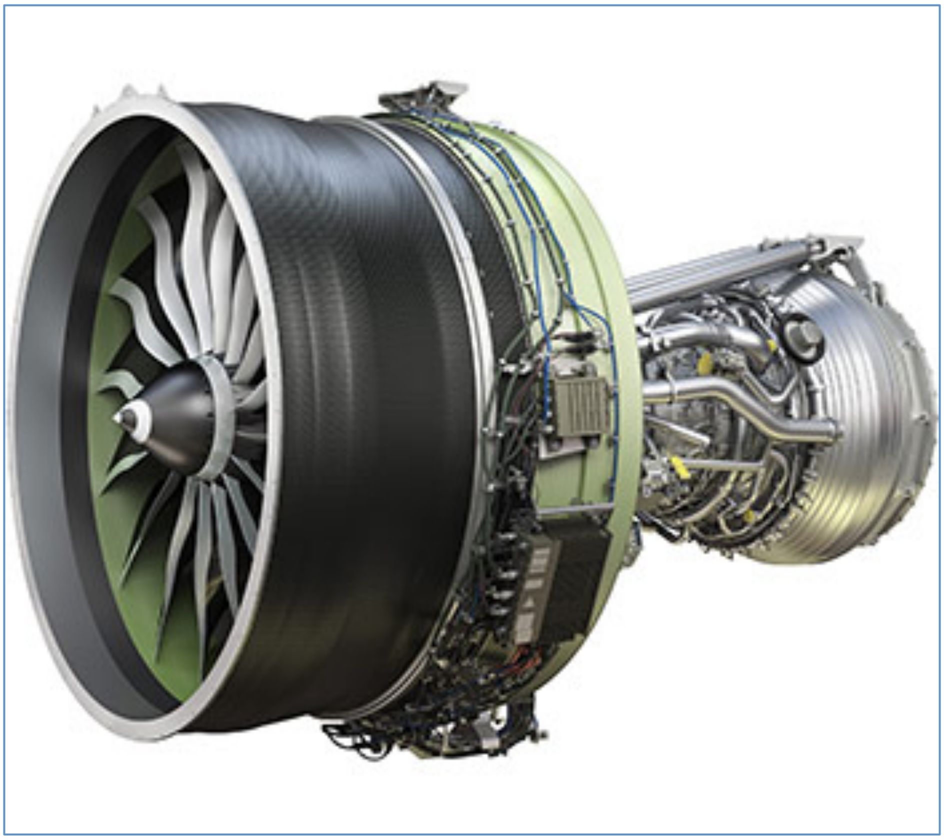

Figure 16: GE9X turbofan engine [geaviation.com]

The Trent 900 turbofan engine generates a thrust output higher than 300 kN, while the GE9X is able to generate up to 440 kN. The turbofan simulated in the Simulink's block system is set to output a maximum of 45 kN of thrust at sea level. Obviously, neither the fan diameter nor the overall pressure ratio should have similar designing parameters to that of the engine developed by Rolls-Royce nor the one constructed by General Electrics.

- **Fan diameter:** to know the volume of gas or working fluid (air) conducted through the core engine, the dimensions of the fan or air inlet diameter of the power plant must be specified. Along with the cruising velocity, this enables the determination of the volume of working fluid treated per unit of time. Today's turbofan engine designs are getting bigger and bigger, because a large inlet drives a higher volume of working fluid through the engine, enabling a higher by-pass rate and thus an improved efficiency. For large commercial airliners, the fan's diameter are usually of around 3 m, as can be noted on the examples presented in the Table 7. To determine this design dimensions a multidomain study has to be performed. As it is not the subject of study of this thesis, the value of the fan diameter will be assumed.

  The maximum power output at sea level of the fanjet subject of simulation is far way lower than the propelling forces generated by those turbofan engines exemplified

(Trent 900 and GE9X). Considering this fact, the value of the fan diameter used for the experimental investigation is set to be of $Ø_i$ = 2m.

Moreover, due to the presence of the fan's blades, as well as of the main shaft running them, the volume of air entering the engine does not correspond to the fan diameter specified. Fan blades occupy between the 5% and 10% of the engine's inlet, while the shaft might occupy between the 10% and 30% of its area. Thus, to drive the calculations, a correction factor of 70% will be used. Applying this factor, the inlet diameter considered to drive the study is of $Ø_i$ = 1,4 m.

- **Overall Pressure Ratio (OPR):** in order to drive the study of the gas turbine cycle, it is necessary to know the pressure relation of the working compressor. Keeping in mind the considerations taken to assume the fan diameter, there is no need for the compressor of the fanjet to raise the pressure of the working fluid to such high values as for the engines exemplified. An OPR greater than a precise amount would withdraw too much mechanical energy out from the turbine, decompensating the engine.

  Obviously, the setting of this parameter should be subject of deep study to meet the most appropriate ratio. Again, as it does not meets the object of this investigation, the OPR is assumed considering the reasons presented. The overall pressure ratio for the compressor subject of further investigation is set to be of OPR = 25:1.

- **Bypass ratio:** as it has been already mentioned, the turbofan simulated is of a high-bypass type, although its ratio has not been specified yet.

  The fanjet subject of study is considered to be used for commercial flights, thus efficiency of operation is of key importance. The higher the by-pass ratio of the engine, the higher will be the efficiency of the turbofan. The core or gas generator of the engine must generate sufficient core power to, at least, drive the fan at its design flow and pressure ratio. Through improvements in turbine cooling/material technology, a higher turbine rotor inlet temperature can be used, thus facilitating a smaller and lighter core. This enables a higher by-pass ratio design, potentially improving the core thermal efficiency.

  Today, as the technology evolves, the bypass ratio for commercial flight's turbofan engines is higher and higher. For the engines specified in the Table 7, the Rolls Royce Trent 900 and the General Electric GE9X have bypass ratio of 8,5-8,7:1 and 9-10,3:1 respectively, although the range varies broadly from 3:1 up to 12:1. The fanjet engine being defined by the simulating program and further specified for a deeper study is smaller and operates generating way less total thrust than the two examples presented. Thus, its bypass ratio, hence its fan thrust, must be also lower. The ratio set for study is of BR = 5:1.

3.1.2. Operating assumptions

The simulator input settings define three operational conditions: the power input or throttle position, the altitude and the speed of flight. The amount of possible combinations within the inputs' values is, let's say, slightly less than countless, and thus, the range of different operating conditions to study boundless for an investigation of this kind.

Most time of a commercial flight is spent while cruising at constant power output settings, speed and altitude. This happens to be somewhere between 9 km and 15 km above the mean sea level. To understand the reason behind it, the simulations will be ran under cruising conditions at different geometrical heights. This is, at the same Mach number and throttle position, while varying the altitude of operation.

In order to perform the study of the gas turbine under the above mentioned circumstances, it is necessary to precise several operating stipulations.

- **Mach number:** airliners propelled by turbofan engines cruise, nowadays, at speeds around M = 0,85. Some of them even reaching and overtaking by few decimals the figure of M = 0,9. As it has already been mentioned, the fanjet engine subject of analysis outputs a thrust force way lower than regular turbofan engines propelling today's commercial aircrafts. However, the speed at cruise of an aircraft depends on many design and operating specifications, but not only on the thrust output of the engine. Thus, a smaller turbofan would be able to drive a smaller aircraft up to speeds as high as greater fanjet engines assembled to larger airliners.

  Still, as the engine being analysed is considered to be used for commercial flights, it might be assumed that this particular design does not reach cruise speeds as high as the two aircraft engines exemplified at Table 7 may reach.

  As an outcome of such deliberation, the Mach number at cruise of the turbofan subject of study is set to be of <u>M = 0,7</u>. The speed corresponding to this at cruising altitudes is approximately of 750 $\frac{km}{h}$.

- **Throttle position:** today's aircrafts have computers, which decide the power settings of the engines given a set of desired flight characteristics by the pilots. This is done in a way that the given specifications are met in the most efficient manner.

  Take-off and landing are the flight stages at which the engines are most demanded. The throttle position at these situations is usually close to 100% or to its maximum opening. On the other hand, while cruising, the power settings are usually set to deliver nearly the maximum thrust possible as well, although because of the lower density of the air at high altitudes, this is far to be as high as at mean-sea level. The decision of the throttle position of an aircraft engine is, as noted, computational, and many variables are considered to meet the most efficient value. However, to achieve high speeds such as M = 0,7 without burning more fuel than needed, it is indispensible to have a large volume of air passing through the engine, so the fan thrust generated is also high.

Taking into consideration all of these factors, the throttle position of the fanjet analysed at cruise is of t_p = 0,8.

- **Fuel type:** in order to drive the study of the gas generator of the turbofan engine it is necessary to know the amount of energy demanded at any moment by its operation. The block system simulating the performance of the power plant outputs the fuel consumption under certain working conditions, although the properties of the fuel used is not specified.

  Illuminating kerosene was used to fuel the first turbine engines. Kerosene is a combustible derived from the distillation crude oil, which is the raw material of the refining industry. There are different types of fuel being used on commercial jet aircrafts, and still today most of them are an heterogeneous mix of some kind based on kerosene. The Jet A and Jet A-1 are the type of fuels most used in aviation. These presented as follows, are their most relevant properties.

	Jet A	**Jet A-1**
Specification	ASTM D 1655	DEF STAN 91-91
Density (15ºC)	775-840 kg/m³	775-840 kg/m³
Flash Point	38 ºC	38 ºC
Freezing Point	-40 ºC	-47 ºC
Net Heat of Combustion	42,8 MJ/kg	42,8 MJ/kg

Table 8: Selected specification properties of jet fuels [Chevron Corporation, 2006]

The Jet A type of fuel is essentially only used in the U.S., while the Jet A-1 type is used in the rest of the world. The greatest differences between the two fuels is that Jet A-1 has a lower maximum freezing point than Jet A. The lower freezing point makes Jet A-1 more suitable for long international flights, especially on polar routes during winter. However, the lower freezing point comes at a price.

Because of the wider application of the Jet A-1 type, this will be the one chosen to drive the study of the simulated turbofan.

3.2. Experimental procedure for the study of a gas turbine cycle

The discussion taking place as follows describes the procedure for the study of the ideal Brayton cycle, which will be applied to the results obtained out from the simulation.

The group of variables outputted by the simulating system are the ones displayed in the next table.

Outputs	**Description**
Thrust	Propelling force generated
Fuel flow	Fuel consumption (kg/s)
P_1	Air pressure at the compressor's inlet
T_1	Air temperature at the compressor's inlet
a	Speed of sound at the operating altitude
ρ	Density of the air at the operating altitude
T_t	Stagnation or total temperature
P_t	Stagnation or total pressure

Table 9: Outputs' short description

In order to describe one by one all the four states of the Brayton cycle, it is intended to compute the mass flow of air treated per second as well as its temperature, pressure and enthalpy at each phase corresponding to the four different states.

The temperature, pressure and density of the air at the first stage, before entering the compressor, are known. To obtain the enthalpy (h) corresponding to the particular state of the air defined by these characters, it is possible to consult an indexed set of tables in which the temperature, the internal energy, the entropy and the mentioned enthalpy are related, describing the state of the air as an ideal gas (Annex A).

Most probably, the enthalpy corresponding to T_1 might not be indexed within this data table, as values are usually not exact. Thus, it is necessary to interpolate the closest known values to it, in order to compute its h (enthalpy).

Applying the Equation 12 for the particular characters subject of study, the desired enthalpy is computed.

To proceed with the study of the Brayton cycle, it is necessary to know the mass of air being analysed. Considering the effects of compressibility of the air at high velocities close to the speed of sound, the relation defining the mass rate of an isentropic flow is the one displayed as follows:

$$\dot{m}_{inlet} = \frac{A_{inlet} \cdot P_t}{\sqrt{T_t}} \cdot \sqrt{\frac{\gamma}{R_{air}}} \cdot M \cdot (1 + \frac{\gamma - 1}{2} \cdot M^2)^{-\frac{(\gamma+1)}{2\cdot(\gamma-1)}}$$

Equation 18

The inlet area is computed under the assumptions set:

$$R_i = \frac{\emptyset_i}{2} = \frac{1,4}{2} = 0,7\ [m]$$

$$A_{inlet} = \pi \cdot R_i^2 = \pi \cdot (0,7)^2 = 1,54\ [m^2]$$

The R_{air} is the specific gas constant for dry air:

$$R_{air} = 287,06\ \frac{J}{kg \cdot K}$$

Thus, the air mass flow entering the engine as a function of the stagnation temperature and pressure, is:

$$\dot{m}_{inlet} = \frac{1,54 \cdot P_t}{\sqrt{T_t}} \cdot \sqrt{\frac{1,4}{287,06}} \cdot 0,7 \cdot (1 + \frac{1,4 - 1}{2} \cdot 0,7^2)^{-\frac{(1,4+1)}{2\cdot(1,4-1)}} = 0,06 \cdot \frac{P_t}{\sqrt{T_t}}$$

The by-pass ratio set for the study is of 5:1. This means 5 kg of air passes around the gas generator through the ducted fan for every 1 kg of air passing through the core engine. Hence, the mass flow of air at the first stage of the Brayton cycle is:

$$\dot{m}_{\text{core}} = \frac{1}{5} \cdot \dot{m}_{inlet}$$

Knowing the compressor pressure ratio (25:1) and the pressure at the inlet of the compressor, it is possible to compute the pressure at its outlet, corresponding to the second state of the air through the Brayton cycle.

$$P_2 = 25 \cdot P_1$$

On the other hand, the following relationship apply for the isentropic processes of compression (stages 1-2) and the decompression that the air undergoes when passing through the gas turbine (stages 3-4).

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma-1}{\gamma})}$$

Equation 19

Again, once the temperature T_2 of the cycle is known, it might be necessary to interpolate in order to obtain the enthalpy corresponding to this state. This is done, as already mentioned, computing the equation 12 for the values of the air characteristics data table closest to the enthalpy desired.

Up to this point, the states one and two of the cycle have been defined.

From state two to three the gas undergoes an isobaric combustion, as the fuel is injected to the working fluid and ignited. For isobaric combustions, the relation displayed below is completed:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o$$

Equation 20

The elements to consider in order to compute the equation above presented are defined as follows:

Approximately, the specific heat capacity of the air at constant pressure and at different temperatures, are:

$$c_{p,air}(-100ºC \text{ to } -50ºC\,) = 1{,}007\ [\frac{kJ}{kg \cdot K}]$$

$$c_{p,air}(-50ºC \text{ to } 0ºC) = 1{,}005\ [\frac{kJ}{kg \cdot K}]$$

The fuel mass flow or fuel consumption outputted by the simulator, under the specified terms, is:

$$\dot{m}_{fuel}$$

The heat of combustion of the fuel, Jet A-1, as noted in the Table 8, is:

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Applying all these characters to the equation described, it is possible to obtain the value of T_3 once it is isolated on the Equation 20.

$$T_3 = \frac{(\dot{m}_{fuel} \cdot \Delta H_c^o) - (\dot{m}_{air} \cdot c_{p,air} \cdot T_2)}{\dot{m}_{core} \cdot c_{p,air}}$$

The enthalpy h_3 is obtained either directly from the data table 'Ideal gas properties of the air' (Annex A) or, again, interpolating.

Moreover, as the combustion is considered to be isobaric, it is true that:

$$P_2 = P_3$$

Now, to proceed defining the temperature at the stage 4, this is after the gas has undergone decompression passing through the turbine, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})}$$

The enthalpy corresponding to the temperature T_4 is obtained as well as has been described previously.

The pressure P_4 is:

$$P_4 = \frac{P_3}{25} = P_1$$

An to end up with the cycle's state description, the volume occupied by the air at the fourth and last stage is computed using the relation displayed at the Equation 18.

$$P_3 \cdot V_3^{\gamma} = P_4 \cdot V_4^{\gamma}; V_4 = \sqrt[\gamma]{\frac{P_3 \cdot V_3^{\gamma}}{P_4}}$$

Along the process described, all the data necessary to compute the power outputted by the engine has been obtained. Applying the following equation, it is attained the power used by the power plant not only to create thrust but to drive the compressor and the fan.

$$\dot{W}_{out} = \dot{m}_{core} \cdot (h_3 - h_4)$$

Equation 21

3.3. Simulation results

The results obtained out from the simulation are presented as a set of outcomes related to the their correspondent inputs. There are three different values to be directly introduced into the computational block: altitude of operation of the engine, Mach number and its throttle position.

The initial thrust, the maximum sea-level thrust, the fastest engine time constant at sea level static, the sea-level static thrust fuel consumption and the ratio of installed to uninstalled thrust parameters are set, as already explained, in the main block's dialog window. The values chosen for each of these parameters are the ones described in the stages 2.5 and 2.6 of the study, and displayed in the figure presented below.

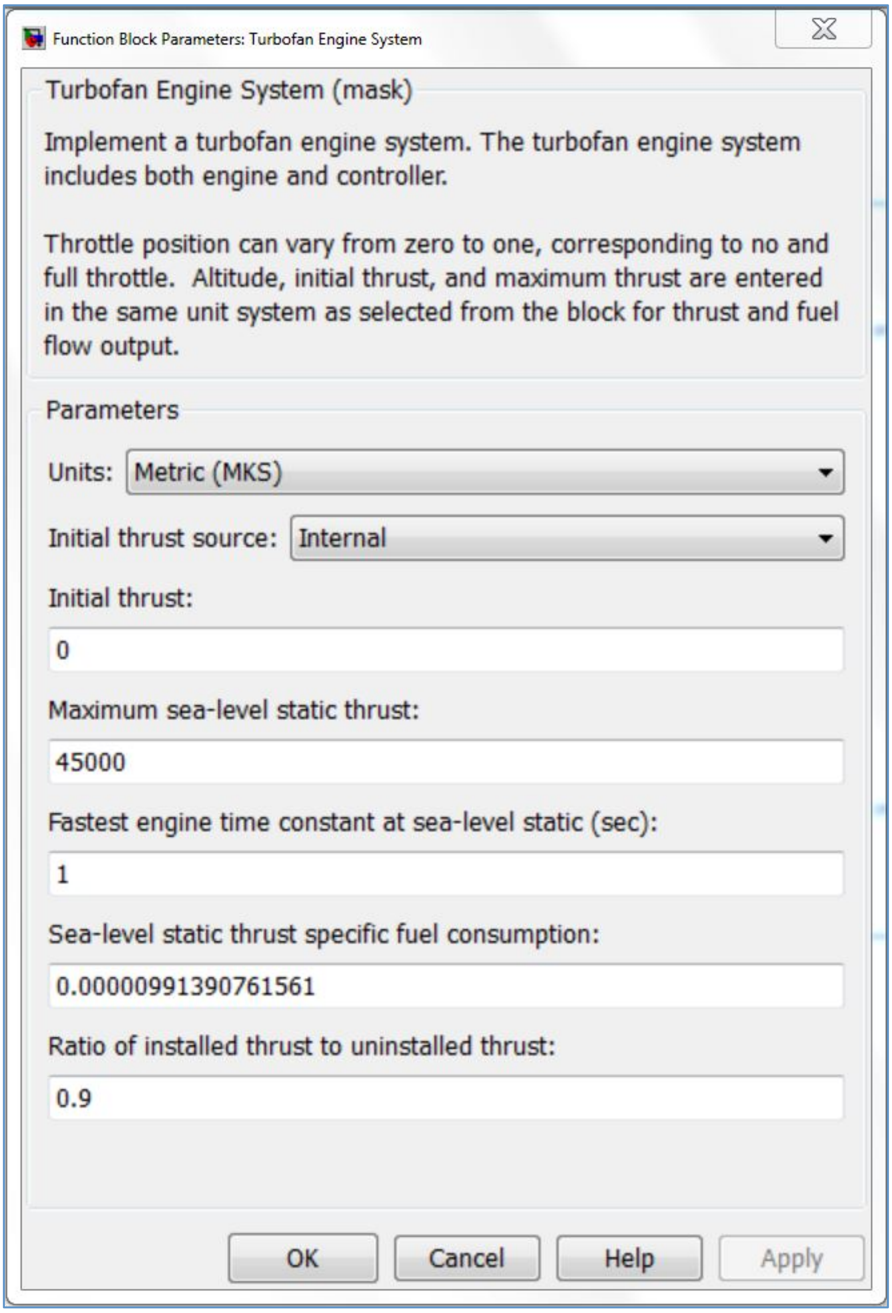

Figure 17: Edited Turbofan Engine System block parameters [MATLAB Simulink]

The simulations' inputs and obtained outputs are presented as follows.

3.3.1. First simulation: altitude = 6000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	$1{,}172 \cdot 10^4$ N
		Fuel flow	0,2544 kg/s
Mach number	0,7	**Atmosphere Pressure**	$4{,}718 \cdot 10^4$ Pa
		Atmosphere Temperature	249,1 K
Altitude	6000 m	**Speed of Sound**	316,4 m/s
		Atmospheric Air Density	0,6597 kg/m^3
		Stagnation Temperature	273,6 K
		Stagnation Pressure	$6{,}544 \cdot 10^4$ Pa

Table 10: First simulation [Author]

3.3.2. Second simulation: altitude = 7000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	$1{,}02 \cdot 10^4$ N
		Fuel flow	0,2184 kg/s
Mach number	0,7	**Atmosphere Pressure**	$4{,}106 \cdot 10^4$ Pa
		Atmosphere Temperature	242,6 K
Altitude	7000 m	**Speed of Sound**	312,3 m/s
		Atmospheric Air Density	0,5895 kg/m^3
		Stagnation Temperature	266,4 K
		Stagnation Pressure	$5{,}696 \cdot 10^4$ Pa

Table 11: Second simulation [Author]

3.3.3. Third simulation: altitude = 8000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	8841 N
		Fuel flow	0,1868 kg/s
Mach number	0,7	**Atmosphere Pressure**	$3,56 \cdot 10^4$ Pa
		Atmosphere Temperature	236,1 K
Altitude	8000 m	**Speed of Sound**	308,1 m/s
		Atmospheric Air Density	0,5252 kg/m^3
		Stagnation Temperature	259,3 K
		Stagnation Pressure	$4,938 \cdot 10^4$

Table 12: Third simulation [Author]

3.3.4. Fourth simulation: altitude = 9000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	7628 N
		Fuel flow	0,1589 kg/s
Mach number	0,7	**Atmosphere Pressure**	$3,074 \cdot 10^4$ Pa
		Atmosphere Temperature	229,6 K
Altitude	9000 m	**Speed of Sound**	303,8 m/s
		Atmospheric Air Density	0,4663 kg/m^3
		Stagnation Temperature	252,2 K
		Stagnation Pressure	$4,264 \cdot 10^4$ Pa

Table 13: Fourth simulation [Author]

3.3.5. Fifth simulation: altitude = 10000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	6566 N
		Fuel flow	0,1349 kg/s
Mach number	0,7	**Atmosphere Pressure**	$2{,}644 \cdot 10^4$ Pa
		Atmosphere Temperature	223,1 K
Altitude	10000 m	**Speed of Sound**	299,5 m/s
		Atmospheric Air Density	0,4127 kg/m^3
		Stagnation Temperature	245 K
		Stagnation Pressure	$3{,}667 \cdot 10^4$ Pa

Table 14: Fifth simulation [Author]

3.3.6. Sixth simulation: altitude = 12000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	4800 N
		Fuel flow	0,09714 kg/s
Mach number	0,7	**Atmosphere Pressure**	$1{,}933 \cdot 10^4$ Pa
		Atmosphere Temperature	216,6 K
Altitude	12000 m	**Speed of Sound**	295,1 m/s
		Atmospheric Air Density	0,3108 kg/m^3
		Stagnation Temperature	237,9 K
		Stagnation Pressure	$2{,}681 \cdot 10^4$ Pa

Table 15: Sixth simulation [Author]

3.3.7. Seventh simulation: altitude = 14000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	3503 N
		Fuel flow	0,07089 kg/s
Mach number	0,7	**Atmosphere Pressure**	$1{,}41 \cdot 10^4$ Pa
		Atmosphere Temperature	216,6 K
Altitude	14000 m	**Speed of Sound**	295,1 m/s
		Atmospheric Air Density	0,2268 kg/m^3
		Stagnation Temperature	237,9 K
		Stagnation Pressure	$1{,}956 \cdot 10^4$ Pa

Table 16: Seventh simulation [Author]

3.3.8. Eighth simulation: altitude = 15000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	2992 N
		Fuel flow	0,06055 kg/s
Mach number	0,7	**Atmosphere Pressure**	$1{,}204 \cdot 10^4$ Pa
		Atmosphere Temperature	216,6 K
Altitude	15000 m	**Speed of Sound**	295,1 m/s
		Atmospheric Air Density	0,1937 kg/m^3
		Stagnation Temperature	237,9 K
		Stagnation Pressure	$1{,}671 \cdot 10^4$ Pa

Table 17: Eighth simulation [Author]

3.3.9. Ninth simulation: altitude = 17000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	2180 N
		Fuel flow	0,04413 kg/s
Mach number	0,7	**Atmosphere Pressure**	8787 Pa
		Atmosphere Temperature	216,6 K
Altitude	17000 m	**Speed of Sound**	295,1 m/s
		Atmospheric Air Density	0,1413 kg/m^3
		Stagnation Temperature	237,9 K
		Stagnation Pressure	$1{,}219 \cdot 10^4$ Pa

Table 18: Ninth simulation [Author]

3.3.10. Tenth simulation: altitude = 20000 m

INPUTS		OUTPUTS	
Throttle	0,8	**Thrust**	1360 N
		Fuel flow	0,02752 kg/s
Mach number	0,7	**Atmosphere Pressure**	5475 Pa
		Atmosphere Temperature	216,6 K
Altitude	20000 m	**Speed of Sound**	295,1 m/s
		Atmospheric Air Density	0,08803 kg/m^3
		Stagnation Temperature	237,9 K
		Stagnation Pressure	7594 Pa

Table 19: Tenth simulation [Author]

3.4. Study of the gas turbine cycle and results

The simulator system outputs the fuel consumption and thrust generated by the described high-bypass turbofan engine, although does not compute the power output of the power plant, this is the energy produced by the gas generator per unit of time. The mechanical work obtained by the turbine set, extracted from the high-energy airflow, is used to drive the compressor and the fan, as well as. Besides, when transformed into electrical energy by a generator, it is also used as a source to run electrical devices of the airplane itself.

The new set of outputs programmed (temperature, pressure and density of the air at the compressor's inlet, as well as the speed of sound) enable a study to achieve a deeper understanding of the core engine's performance while cruising at different altitudes.

As follows are presented the results of the analysis performed to the core engine of the computer-generated turbofan. These are displayed simulation-by-simulation. The Brayton cycle corresponding to each simulation ran is defined, stage-by-stage, by the outcomes of the previously detailed experimental procedure.

3.4.1. Gas turbine cycle at altitude = 6000 m

Brayton cycle's stages	**Temperature (K)**	**Pressure (Pa)**	**Enthalpy (kJ/kg)**	**Mass flow (kg/s)**
1	249,1	$4{,}72 \cdot 10^4$	249,15	44,98
2	624,86	$1{,}18 \cdot 10^6$	633,21	
3	865,72	$1{,}18 \cdot 10^6$	894,65	
4	345,12	$4{,}72 \cdot 10^4$	345,57	
Power output (kW)	24698,05			

Table 20: Gas turbine cycle's results corresponding to the first simulation [Author]

3.4.2. Gas turbine cycle at altitude = 7000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	242,6	$4{,}11 \cdot 10^4$	242,63	39,68
2	608,56	$1{,}03 \cdot 10^6$	616,01	
3	842,97	$1{,}03 \cdot 10^6$	869,38	
4	336,05	$4{,}11 \cdot 10^4$	336,44	
Power output (kW)	21145,96			

Table 21: Gas turbine cycle's results corresponding to the second simulation [Author]

3.4.3. Gas turbine cycle at altitude = 8000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	236,1	$3{,}56 \cdot 10^4$	236,12	34,86
2	592,25	$8{,}90 \cdot 10^5$	598,88	
3	820,42	$8{,}90 \cdot 10^5$	844,45	
4	327,06	$3{,}56 \cdot 10^4$	327,38	
Power output (kW)	18027,67			

Table 22: Gas turbine cycle's results corresponding to the third simulation [Author]

3.4.4. Gas turbine cycle at altitude = 9000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	229,6	$3{,}07 \cdot 10^4$	229,62	30,53
2	575,95	$7{,}68 \cdot 10^5$	581,81	
3	797,62	$7{,}68 \cdot 10^5$	819,34	
4	317,97	$3{,}07 \cdot 10^4$	318,25	
Power output (kW)	15297,09			

Table 23: Gas turbine cycle's results corresponding to the fourth simulation [Author]

3.4.5. Gas turbine cycle at altitude = 10000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	223,1	$2{,}64 \cdot 10^4$	223,08	26,64
2	559,64	$6{,}61 \cdot 10^5$	564,79	
3	774,89	$6{,}61 \cdot 10^5$	794,46	
4	308,91	$2{,}64 \cdot 10^4$	309,14	
Power output (kW)	12926,94			

Table 24: Gas turbine cycle's results corresponding to the fifth simulation [Author]

3.4.6. Gas turbine cycle at altitude = 12000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	216,6	$1,93 \cdot 10^4$	216,57	19,7627
2	543,34	$4,83 \cdot 10^5$	547,82	
3	752,25	$4,83 \cdot 10^5$	769,74	
4	299,88	$1,93 \cdot 10^4$	300,07	
Power output (kW)	9281,92			

Table 25: Gas turbine cycle's results corresponding to the sixth simulation [Author]

3.4.7. Gas turbine cycle at altitude = 14000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	216,6	$1,41 \cdot 10^4$	216,57	14,42
2	543,34	$3,53 \cdot 10^5$	547,82	
3	752,31	$3,53 \cdot 10^5$	769,80	
4	299,90	$1,41 \cdot 10^4$	300,09	
Power output (kW)	6520,18			

Table 26: Gas turbine cycle's results corresponding to the seventh simulation [Author]

3.4.8. Gas turbine cycle at altitude = 15000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	216,6	$1,20 \cdot 10^4$	216,57	12,32
2	543,34	$3,01 \cdot 10^5$	547,82	
3	752,27	$3,01 \cdot 10^5$	769,76	
4	299,89	$1,20 \cdot 10^4$	300,08	
Power output (kW)	5785,34			

Table 27: Gas turbine cycle's results corresponding to the eighth simulation [Author]

3.4.9. Gas turbine cycle at altitude = 17000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	216,6	8787	216,57	8,98
2	543,34	$2,20 \cdot 10^5$	547,82	
3	752,07	$2,20 \cdot 10^5$	769,55	
4	299,8116	8787	300,00	
Power output (kW)	4219,21			

Table 28: Gas turbine cycle's results corresponding to the ninth simulation [Author]

3.4.10. Gas turbine cycle at altitude = 20000 m

Brayton cycle's stages	Temperature (K)	Pressure (Pa)	Enthalpy (kJ/kg)	Mass flow (kg/s)
1	216,6	5475	216,57	5,5978
2	543,34	$1{,}37 \cdot 10^5$	547,82	
3	752,29	$1{,}37 \cdot 10^5$	769,78	
4	299,89	5475	300,08	
Power output (kW)	2629,27			

Table 29: Gas turbine cycle's results corresponding to the tenth simulation [Author]

3.5. Results discussion

The results obtained show that raising the altitude of operation drives into a reduction of fuel consumption, although it comes with a consequent decrease of thrust force generated, as well as a lower sum of mechanical power outputted by the turbine.

Considering the kilometres covered per unit mass of fuel consumed, it is obvious that at higher altitudes the propulsion system performs more efficiently. However, this comes with a detriment in the cruising velocity, as the Mach number remains constant while the speed of sound decrease when raising the operational height.

While at low altitudes (6000 m above mean sea level) the propulsion system covers slightly less than one kilometre per kilogram of fuel burnt, its efficiency rises exponentially as the altitude of operation is lifted up: cruising at 20000 m above the mean se level, the engine would cover more than seven kilometres per kilogram of fuel consumed.

The cause of the improved propulsion efficiency of the system at high altitudes is, mainly, the lower density of the air. Because of this, the vehicle finds less aerodynamic resistance and the drag force generated by its displacement is lower. Hence, the system covers larger distances with smaller propelling or thrust force, as it diminishes at high operational heights.

In the figure presented below, a graph displays how many kilometres does the system fly by per unit mass of fuel used, at different altitudes. The representation is bounded within the range of altitudes comprised along the simulation.

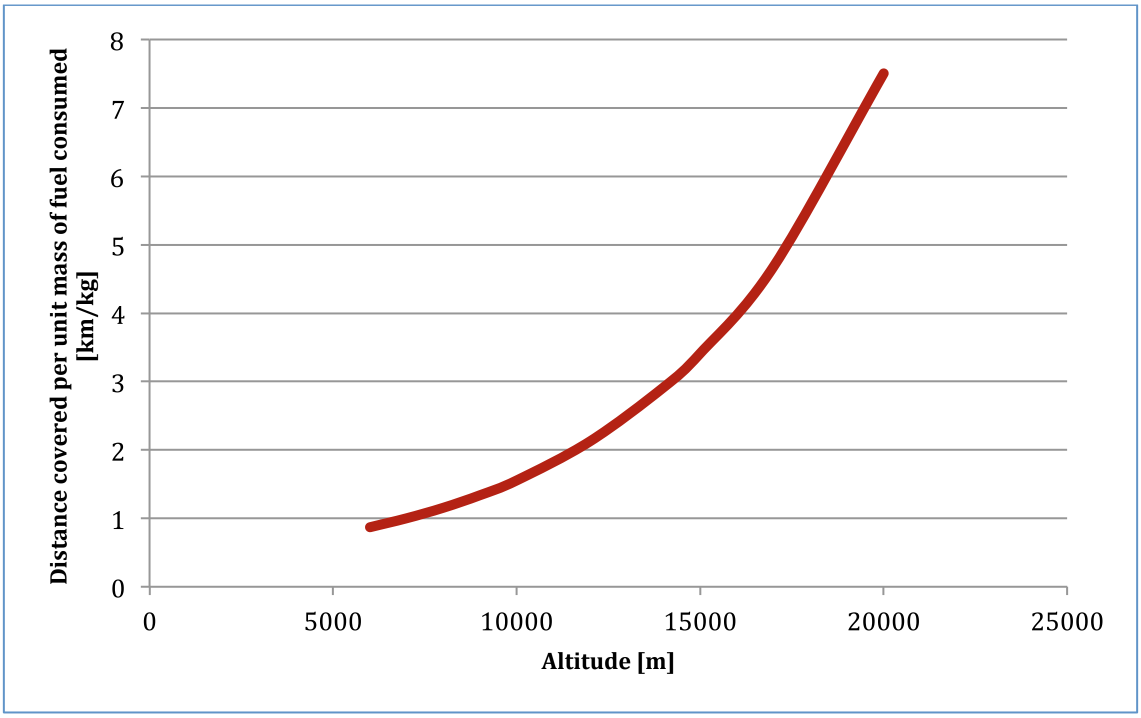

Figure 18: Distance covered per unit mass of fuel consumed at different altitudes

On the other hand, if the core engine's efficiency is considered, the energy or work produced by the turbine set as the working fluid undergoes the Brayton cycle has to be compared to the mass of fuel consumed along the process.

The figure below displays a graph representing the work or energy produced by the core engine per unit mass of fuel consumed, at different altitudes. Again, the representation is bounded within the range of altitudes comprised along the simulation.

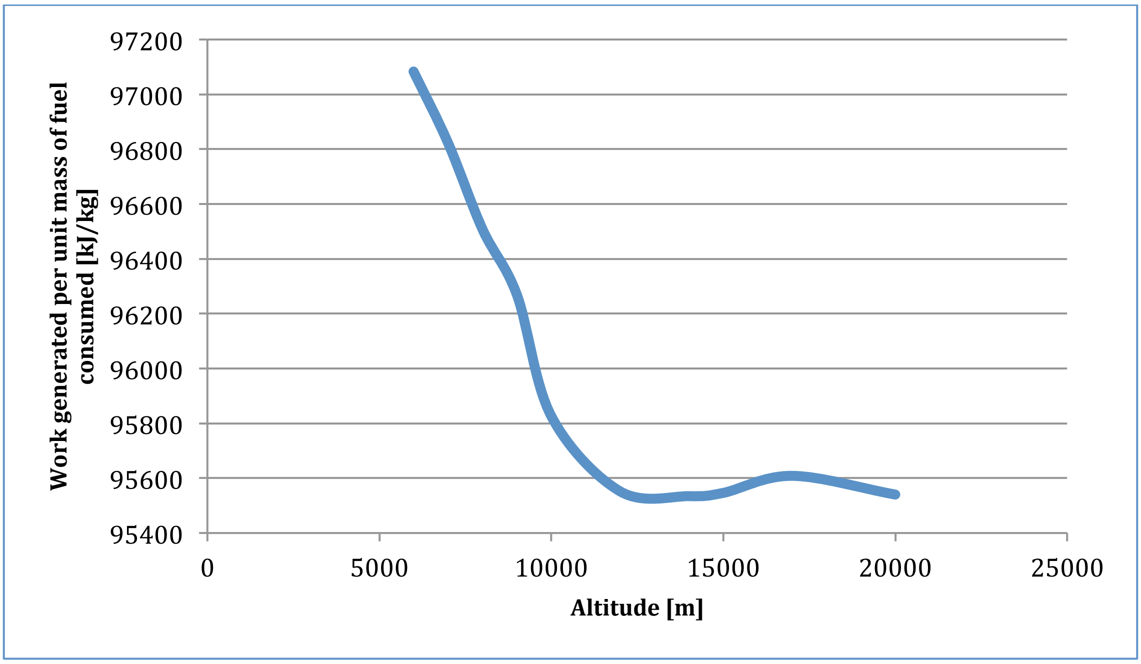

Figure 19: Work generated per unit mass of fuel consumed at different altitudes

As it is shown in the Figure 7, the efficiency of the core engine diminishes at high altitudes. The decrease is remarkable at first, between 6000 m and 12000 m and it is later stabilized. This particular behaviour corresponds to a higher rate of change of the conditions or air characteristics at lower altitudes, while the air temperature, pressure and density do not vary so steeply at higher operational heights.

4. Budget

The budget of this project is separated in three parts: personnel costs, software acquisition and other costs.

4.1. Personnel costs

To calculate personnel costs, the price per hour that an engineer student costs to a company as an intern is of 9,58 €, including indirect costs.

Personnel costs				
Phase	*Description*	*Quantity*	*Unitary cost*	*Cost*
Preparation	Bibliographic research	80 h	9,58 €	766,4 €
Simulation	Simulation model groundwork	90 h	9,58 €	862,2 €
	Simulation model edition	115 h	9,58 €	1101,7 €
	Simulation data post-processing	230 h	9,58 €	2203,4 €
Thesis	Wording	250 h	9,58 €	2395 €
Total				7328,7 €

Table 30: Personnel costs [Author]

4.2. Software acquisition costs

Simulation costs				
Phase	*Description*	*Quantity*	*Unitary cost*	*Cost*
MATLAB Simulink	Simulating software	1 license	0 €	0 €
Microsoft Office	Micorsoft Office pack	1 license	119 €	119 €
Total				119 €

Table 31: Simulation acquisition costs [Author]

Mathworks provides free annual MATLAB Simulink educational licences for students around the world. Thus the software used to study and predict the behaviour of a turbofan engine has resulted costless.

4.3. Other costs

Other costs				
Phase	*Description*	*Quantity*	*Unitary cost*	*Cost*
Traveling	Flight Stuttgart-Barcelona	1 ticket	107,49 €	107,49 €
	Flight Barcelona-Stuttgart	1 ticket	62,49 €	62,49 €
Bookbinding	Memory printing and bookbinding	1 memory	112,40 €	112,40 €
Total				282,38 €

Table 32: Other costs [Author]

4.4. Total costs

Total costs	
Type	*Cost*
Personnel	7328,7 €
Software acquisition	119 €
Others	282,38 €
Total	7730,08 €

Table 33: Total costs [Author]

5. Conclusions

Along the development of this thesis, the efficiency of airliners' flights has been questioned from both a technical and operational point of view. To do so, the study of the operational principles and environment, as well as of the turbofans' state of the art has been developed aside with the research of a simulating system, which has enabled the prediction of the behaviour of a particularly defined fanjet while cruising at different altitudes.

Up to this stage of the investigation, it is obvious that depending on the altitude of flight when cruising, turbofan engines will perform consuming different amounts of fuel. Considering the great amount of kerosene burnt by turbofans in order to propel aircrafts, coupled with the huge volume of departures scheduled day by day, it is of key importance for the society and the environment to study how to limit the use of fossil fuels without compromising progress. Hence, while no clean technologies are enough developed to substitute the actual aircraft propulsion systems, it is compulsory to improve at its maximum its efficiency so it is possible to diminish the impact of air transports on the planet earth.

Turbofan engines, as any other type of jet engine, will perform more resourcefully at high altitudes (Figure 16). The upper limit to that might be of different natures:

- **Length of flight:** the most fuel demanding phases of a flight are both take-off, specially, and landing. Therefore the optimum altitude of flight will not be the same for a regional airliner as for a commercial transatlantic plane. The cruising height for the first one would be lower, as climbing up to higher layers of the atmosphere, where the engine's performance is more efficient, might not be worth it due to the relatively shorter time of flight. Thus, the altitude of cruise must be calculated as a function of time and fuel sources necessary to achieve that particular height, as well as of the overall time of flight.
- **Core engine's performance:** conversely to the propelling efficiency, the amount of work generated by the turbine per unit mass of fuel used is lower at high altitudes. The energy generated by the turbine will be used in a wide group of applications: it runs the compressor and fan of the engine itself; powers the electrical devices of the vehicle; drives the hydraulic pumps; actuate the landing gear, as well as any of the extra energy depending devices of the aircraft.

In order to meet the most efficient altitude of cruise, it is essential to consider the minimum amount of mechanical energy outputted by the turbine necessary to run all the auxiliary units of the aircraft. And this, to be related with the above mentioned stipulations regarding the length of flight.

Many other specifications bound the range of altitudes at which an aircraft might cruise, and a great part of them are security stipulations. Taking all of them into account would be possible to define an optimum route, strictly defined for each aircraft model and its particular flight conditions. The path to achieve so is to create a function contemplating all of these factors, pondered by its influence on the efficiency of flight. Here, humbly, the definition of two of these factors have been pursued.

6. Future work

Future work could be focused on simulating specific turbofan engine designs, using FEM analysis. For a particular fanjet, considering all of its design and operational characteristics, as well as defining explicit working conditions regarding the aircraft to which it is coupled, would be possible to describe a prime route, in a more strict way in which today's flight itineraries are defined. Besides, the security stipulations would have to be considered, relating them to the equation formed by all the previously mentioned factors.

The addition of these to the considerations discussed and presented within this investigation paper, might reduce the fuel consumption. Improving the efficiency and diminishing costs of operation could enable the cheapening of flying, so more people could afford it.

However, this should be only an intermediate step into a complete carbon-free propulsion system, which would minimize the impact on the environment without compromising progress.

7. Literature

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ANNEX I

Ideal Gas Properties Of The Air

T(K), h and u (kJ/kg), s (kJ/kg·K)							
T	h	u	s	T	h	u	s
200	199,97	142,56	1,29559	510	513,32	366,92	2,23993
210	209,97	149,69	1,34444	520	523,63	374,36	2,25997
220	219,97	156,82	1,39105	530	533,98	381,84	2,27967
230	230,02	164	1,43557	540	544,35	389,34	2,29906
240	240,02	171,13	1,47824	550	554,74	396,86	2,31809
250	250,05	178,28	1,51917	560	565,17	404,42	2,33685
260	260,09	185,45	1,55848	570	575,59	411,97	2,35531
270	270,11	192,6	1,59634	580	586,04	419,55	2,37348
280	280,13	199,75	1,63279	590	596,52	427,15	2,39140
290	290,16	206,91	1,66802	600	607,02	434,78	2,39140
300	300,19	214,07	1,70203	610	617,53	442,42	2,42644
310	310,24	221,25	1,73498	620	628,07	450,09	2,44356
320	320,29	228,42	1,76690	630	638,63	457,78	2,46048
330	330,34	235,61	1,79783	640	649,22	465,5	2,47716
340	340,42	242,82	1,82790	650	659,84	473,25	2,49364
350	350,49	250,02	1,85708	660	670,47	481,01	2,50985
360	360,58	257,24	1,88543	670	681,14	488,81	2,52589
370	370,67	264,46	1,91313	680	691,82	496,62	2,54175
380	380,77	271,69	1,94001	690	702,52	504,45	2,55731
390	390,88	278,93	1,96633	700	713,27	512,33	2,57277
400	400,98	286,16	1,99194	710	724,04	520,23	2,58810
410	411,12	293,43	2,01699	720	734,82	528,14	2,60319
420	421,26	300,69	2,04142	730	745,62	536,07	2,61803
430	431,43	307,99	2,06533	740	756,44	544,02	2,6328
440	441,61	315,30	2,08870	750	767,29	551,99	2,64737
450	451,80	322,62	2,11161	760	778,18	560,01	2,66176
460	462,02	329,97	2,13407	770	789,11	568,07	2,67595
470	472,24	337,32	2,15604	780	800,03	576,12	2,69013
480	482,49	344,70	2,17760	790	810,99	584,21	2,704
490	492,74	352,08	2,19876	800	821,95	592,30	2,71787
500	503,02	359,49	2,21952	820	843,98	608,59	2,74504

T(K), h and u (kJ/kg), s (kJ/kg·K)							
T	*h*	*u*	*s*	*T*	*h*	*u*	*s*
840	866,08	624,95	2,7717	1420	1539,44	1131,77	3,37901
860	888,27	641,4	2,79783	1440	1563,51	1150,13	3,39586
880	910,56	657,95	2,82344	1460	1587,63	1168,49	3,41247
900	932,93	674,58	2,84856	1480	1611,79	1186,95	3,42892
920	955,38	691,28	2,87324	1500	1635,97	1205,41	3,44516
940	977,92	708,08	2,89748	1520	1660,23	1223,87	3,4612
960	1000,55	725,02	2,92128	1540	1684,51	1242,43	3,47712
980	1023,25	741,98	2,94468	1560	1708,82	1260,99	3,49276
1000	1046,04	758,94	2,9677	1580	1733,17	1279,65	3,50829
1020	1068,89	776,1	2,99034	1600	1757,57	1298,3	3,52364
1040	1091,85	793,36	3,01260	1620	1782	1316,96	3,53879
1060	114,86	810,62	3,03449	1640	1806,46	1335,72	3,55381
1080	1137,89	827,88	3,05608	1660	1830,96	1354,48	3,56867
1100	1161,07	845,33	3,07732	1680	1855,5	1373,24	3,58335
1120	1184,28	862,79	3,09825	1700	1880,1	1392,7	3,5979
1140	1207,57	880,35	3,11883	1750	1941,6	1439,8	3,6336
1160	1230,92	897,91	3,13916	1800	2003,3	1487,2	3,6684
1180	1254,34	915,57	3,15916	1850	2065,3	1534,9	3,7023
1200	1277,79	933,33	3,17888	1900	2127,4	1582,6	3,7354
1220	1301,79	951,09	3,19834	1950	2189,7	1630,6	3,7677
1240	1324,93	968,95	3,21751	2000	2252,1	1678,7	3,7994
1260	1348,55	986,90	3,23638	2050	2314,6	1726,8	3,8303
1280	1372,24	1004,76	3,25510	2100	2377,4	1775,3	3,8605
1300	1395,97	1022,82	3,27345	2150	2440,3	1823,8	3,8901
1320	1419,76	1040,88	3,29160	2200	2503,2	1872,4	3,9191
1340	1443,60	1058,94	3,30959	2250	2566,4	1921,3	3,9474
1360	1467,49	1077,1	3,32724				
1380	1491,44	1095,26	3,34474				
1400	1515,42	1113,52	3,362				

Table 34: Gas Tables [Keenan, J. H., Kaye, J., 1945]

ANNEX II

Calculations

a. First simulation

As the temperature T_1 is known (T_1 = 249,1 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 240 K; h_- = 240,02 kJ/kg

T_+ = 250 K; h_+ = 250,05 kJ/kg

$$h_1 = 240{,}02 + (250{,}05 - 240{,}02) \cdot \frac{(249{,}02-240)}{(250-240)} = 249{,}1473\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 6000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{6{,}544 \cdot 10^4}{\sqrt{273{,}6}} = 224{,}9062\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{224{,}9062}{5} = 44{,}9812\ \frac{\mathrm{kg}}{s}$$

Considering $P_1 = 4{,}7180 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 4{,}7180 \cdot 10^4 = 1{,}1795 \cdot 10^6\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = 249{,}1 \cdot (25)^{\left(\frac{1{,}4-1}{1{,}4}\right)} = 624{,}8635\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

T_- = 620 K; h_- = 628,07 kJ/kg

T_+ = 630 K; h_+ = 638,63 kJ/kg

$$h_2 = 628{,}07 + (638{,}63 - 628{,}07) \cdot \frac{(624{,}8635 - 620)}{(630 - 620)} = 633{,}2059\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 44{,}9812\ \frac{\text{kg}}{s}$$

$$c_{p,air}(-50^{\text{o}}\text{C to } 0^{\text{o}}\text{C}) = 1{,}005\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}2544\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$44{,}9812 \cdot 1{,}005 \cdot (T_3 - 624{,}8635) = 0{,}2544 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{10888{,}32 + 28247{,}6695}{45{,}2061} = 865{,}7228\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 860$ K; $h_- = 888{,}27$ kJ/kg

$T_+ = 880$ K; $h_+ = 910{,}56$ kJ/kg

$$h_3 = 888{,}27 + (910{,}56 - 888{,}27) \cdot \frac{(865{,}7228 - 860)}{(880 - 860)} = 894{,}6481\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 1{,}1795 \cdot 10^6\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 4{,}7180 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 865{,}7228 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 345{,}1178\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 340$ K; $h_- = 340{,}42$kJ/kg

$T_+ = 350$ K; $h_+ = 350{,}49$ kJ/kg

$$h_4 = 340{,}42 + (350{,}49 - 340{,}42) \cdot \frac{(345{,}1178 - 340)}{(350 - 340)} = 345{,}5737 \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 44{,}9812 \cdot (894{,}6481 - 345{,}5737) = 24698{,}0454\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 316{,}4 = 221{,}48\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2215\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2215}{0{,}2544} = 0{,}8706 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{24698{,}0454}{0{,}2544} = 97083{,}5117\ \frac{kJ}{kg}$$

b. Second simulation

As the temperature T_1 is known (T_1 = 242,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 240 K; h_- = 240,02 kJ/kg

T_+ = 250 K; h_+ = 250,05 kJ/kg

$$h_1 = 240{,}02 + (250{,}05 - 240{,}02) \cdot \frac{(242{,}06 - 240)}{(250 - 240)} = 242{,}6278 \ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 7000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{5{,}696 \cdot 10^4}{\sqrt{266{,}4}} = 198{,}3897 \ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{198{,}3897}{5} = 39{,}6779 \ \frac{\mathrm{kg}}{s}$$

Considering $P_1 = 4{,}106 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 4{,}106 \cdot 10^4 = 1{,}0265 \cdot 10^6 \ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma - 1}{\gamma}\right)} = 242{,}6 \cdot (25)^{\left(\frac{1{,}4-1}{1{,}4}\right)} = 608{,}5583 \ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

T_- = 600 K; h_- = 607,02 kJ/kg

T_+ = 610 K; h_+ = 617,53 kJ/kg

$$h_2 = 607{,}02 + (617{,}53 - 607{,}02) \cdot \frac{(608{,}5583 - 600)}{(610 - 600)} = 616{,}0148 \ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 39{,}6779 \ \frac{kg}{s}$$

$$c_{p,air}(-50ºC \text{ to } 0ºC) = 1{,}005 \ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}2184 \ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$39{,}6779 \cdot 1{,}005 \cdot (T_3 - 608{,}5583) = 0{,}2184 \cdot (42{,}8 \cdot 10^3); \ T_3 = \frac{9347{,}52 + 24267{,}0686}{39{,}8763} =$$

$$842{,}9711 \ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 840$ K; $h_- = 866{,}08$ kJ/kg

$T_+ = 860$ K; $h_+ = 888{,}27$ kJ/kg

$$h_3 = 866{,}08 + (888{,}27 - 866{,}08) \cdot \frac{(842{,}9711)}{(860 - 840)} = 869{,}3765 \ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 1{,}0265 \cdot 10^6 \ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 4{,}106 \cdot 10^4 \ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma - 1}{\gamma})} = 869{,}3765 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 336{,}0479 \ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 330$ K; $h_- = 330{,}34$ kJ/kg

$T_+ = 340$ K; $h_+ = 340{,}42$ kJ/kg

$$h_4 = 340{,}42 + (330{,}34 - 340{,}42) \cdot \frac{(336{,}0479 - 330)}{(340 - 330)} = 336{,}4364\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 39{,}6779 \cdot (869{,}3765 - 336{,}4364) = 21145{,}9623\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 312{,}3 = 218{,}61\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2186\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2186}{0{,}2184} = 1{,}0009 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{21145{,}9623}{0{,}2184} = 96822{,}1718\ \frac{kJ}{kg}$$

c. Third simulation

As the temperature T_1 is known (T_1 = 236,1 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

$T_- = 230$ K; $h_- = 230{,}02$ kJ/kg

$T_+ = 240$ K; $h_+ = 240{,}02$ kJ/kg

$$h_1 = 230{,}02 + (240{,}02 - 230{,}02) \cdot \frac{(236{,}1-230)}{(240-230)} = 236{,}12\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 8000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{4{,}938 \cdot 10^4}{\sqrt{259{,}3}} = 174{,}3275\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{174{,}3275}{5} = 34{,}8655\ \frac{\mathrm{kg}}{s}$$

Considering $P_1 = 3{,}56 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 3{,}56 \cdot 10^4 = 8{,}9 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma-1}{\gamma})} = 236{,}1 \cdot (25)^{(\frac{1{,}4-1}{1{,}4})} = 592{,}2532\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

$T_- = 590$ K; $h_- = 596{,}52$ kJ/kg

$T_+ = 600$ K; $h_+ = 607{,}02$ kJ/kg

$$h_2 = 596{,}52 + (607{,}02 - 596{,}52) \cdot \frac{(592{,}2532 - 590)}{(600 - 590)} = 598{,}8859\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 34{,}8655\ \frac{kg}{s}$$

$$c_{p,air}(-50ºC \text{ to } 0ºC) = 1{,}005\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}1868\frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3[\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$34{,}8655 \cdot 1{,}005 \cdot (T_3 - 592{,}2532) = 0{,}1868 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{7995{,}04+20752{,}4501}{35{,}0398} = 820{,}4233\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 820$ K; $h_- = 843{,}98$ kJ/kg

$T_+ = 840$ K; $h_+ = 866{,}08$ kJ/kg

$$h_3 = 843{,}98 + (866{,}08 - 843{,}98) \cdot \frac{(820{,}4233 - 820)}{(840 - 820)} = 844{,}4477\frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 8{,}9 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 3{,}56 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 820{,}4233 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 327{,}0593\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 320$ K; $h_- = 320{,}29$ kJ/kg

$T_+ = 330$ K; $h_+ = 330{,}34$ kJ/kg

$$h_4 = 320{,}29 + (330{,}34 - 320{,}29) \cdot \frac{(327{,}0593 - 320)}{(330 - 320)} = 327{,}3846\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 34{,}3846 \cdot (844{,}4477 - 327{,}3846) = 18027{,}6669\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 308{,}1 = 215{,}67\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2157\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2157}{0{,}1868} = 1{,}1545 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{18027{,}6669}{0{,}1868} = 96507{,}8531 \frac{kJ}{kg}$$

d. Fourth simulation

As the temperature T_1 is known (T_1 = 229,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 220 K; h_- = 219,97 kJ/kg

T_+ = 230 K; h_+ = 230,02 kJ/kg

$$h_1 = 219{,}97 + (230{,}02 - 219{,}97) \cdot \frac{(229{,}6-220)}{(230-220)} = 229{,}618\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 9000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{4{,}264 \cdot 10^4}{\sqrt{252{,}2}} = 152{,}6374 \ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{152{,}6374}{5} = 30{,}5275 \ \frac{\text{kg}}{s}$$

Considering $P_1 = 3{,}074 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 3{,}0740 \cdot 10^4 = 7{,}6850 \cdot 10^5 \ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma - 1}{\gamma})} = 229{,}6 \cdot (25)^{(\frac{1{,}4-1}{1{,}4})} = 575{,}9480 \ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

$T_- = 570$ K; $h_- = 575{,}59$ kJ/kg

$T_+ = 580$ K; $h_+ = 586{,}04$ kJ/kg

$$h_2 = 575{,}59 + (586{,}04 - 575{,}59) \cdot \frac{(575{,}9480 - 570)}{(580 - 570)} = 581{,}8057 \ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 30{,}5275 \frac{kg}{s}$$

$$c_{p,air}(-50ºC \text{ to } 0ºC) = 1{,}005 \ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}1589 \ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$530{,}5275 \cdot 1{,}005 \cdot (T_3 - 575{,}9480) = 0{,}1589 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{6800{,}92+17670{,}14811}{30{,}6801} = 797{,}6200\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 790$ K; $h_- = 810{,}99$ kJ/kg

$T_+ = 800$ K; $h_+ = 821{,}95$ kJ/kg

$$h_3 = 810{,}99 + (821{,}95 - 810{,}99) \cdot \frac{(797{,}62 - 790)}{(800 - 790)} = 819{,}3415 \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 7{,}6850 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 3{,}0740 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 819{,}3415 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 317{,}9689\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 310$ K; $h_- = 310{,}24$ kJ/kg

$T_+ = 320$ K; $h_+ = 320{,}29$ kJ/kg

$$h_4 = 310{,}24 + (320{,}24 - 310{,}24) \cdot \frac{(317{,}9689 - 310)}{(320 - 310)} = 318{,}2487\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 530{,}5275 \cdot (819{,}3415 - 318{,}2487) = 15297{,}0965\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 303{,}8 = 212{,}66\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2127\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2127}{0{,}1589} = 1{,}3383 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{15297{,}0965}{0{,}1589} = 96268{,}7007 \frac{kJ}{kg}$$

e. Fifth simulation

As the temperature T_1 is known (T_1 = 223,1 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 220 K; h_- = 219,97 kJ/kg

T_+ = 230 K; h_+ = 230,02 kJ/kg

$$h_1 = 219{,}97 + (230{,}02 - 219{,}97) \cdot \frac{(223{,}1 - 220)}{(230 - 220)} = 223{,}0855\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 10000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{3{,}667 \cdot 10^4}{\sqrt{245}} = 133{,}1815\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{133{,}1815}{5} = 26{,}6363\ \frac{\text{kg}}{s}$$

Considering $P_1 = 2{,}644 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 2{,}644 \cdot 10^4 = 6{,}61 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma-1}{\gamma})} = 223{,}1 \cdot (25)^{(\frac{1{,}4-1}{1{,}4})} = 559{,}6429\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

$T_- = 550$ K; $h_- = 554{,}74$ kJ/kg

$T_+ = 560$ K; $h_+ = 565{,}17$ kJ/kg

$$h_2 = 554{,}74 + (565{,}17 - 554{,}74) \cdot \frac{(559{,}6429 - 550)}{(560 - 550)} = 564{,}7975\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 26{,}6363\ \frac{kg}{s}$$

$$c_{p,air}(-100ºC \text{ to } -50ºC) = 1{,}007\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}1349\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$26{,}6363 \cdot 1{,}007 \cdot (T_3 - 559{,}6429) = 0{,}1349 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{5773{,}72+15011{,}1639}{26{,}8227} =$$

$$774{,}8975\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 770\ K;\ h_- = 789{,}11\ kJ/kg$

$T_+ = 780\ K;\ h_+ = 800{,}03\ kJ/kg$

$$h_3 = 789{,}11 + (800{,}03 - 789{,}11) \cdot \frac{(774{,}8975 - 770)}{(780 - 770)} = 794{,}4580\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 6{,}61 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 2{,}644 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 774{,}8975 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 308{,}9106\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 300\ K;\ h_- = 300{,}19\ kJ/kg$

$T_+ = 310\ K;\ h_+ = 310{,}24\ kJ/kg$

$$h_4 = 300{,}19 + (310{,}24 - 300{,}19) \cdot \frac{(308{,}9106 - 300)}{(310 - 300)} = 309{,}1451\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 26{,}6363 \cdot (794{,}4580 - 309{,}1451) = 12926{,}9431\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 299{,}5 = 209{,}65\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2096\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2096}{0{,}1349} = 1{,}5541\,\frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{12926{,}9431}{0{,}1349} = 95826{,}1163\,\frac{kJ}{kg}$$

f. Sixth simulation

As the temperature T_1 is known (T_1 = 216,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 210 K; h_- = 209,97 kJ/kg

T_+ = 220 K; h_+ = 219,97 kJ/kg

$$h_1 = 209{,}97 + (219{,}97 - 209{,}97) \cdot \frac{(216{,}6-210)}{(220-210)} = 216{,}57\,\frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 12000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{2{,}681 \cdot 10^4}{\sqrt{237{,}9}} = 98{,}8134\,\frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{98{,}8134}{5} = 19{,}7627\,\frac{\text{kg}}{s}$$

Considering $P_1 = 1{,}933 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 1{,}933 \cdot 10^4 = 4{,}8325 \cdot 10^5\,Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = 216{,}6 \cdot (25)^{\left(\frac{1{,}4-1}{1{,}4}\right)} = 543{,}3377\,K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

$T_- = 540$ K; $h_- = 544{,}35$ kJ/kg

$T_+ = 550$ K; $h_+ = 554{,}74$ kJ/kg

$$h_2 = 544{,}35 + (554{,}74 - 544{,}35) \cdot \frac{(543{,}3377 - 540)}{(550 - 540)} = 547{,}8179\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 19{,}7627 \frac{kg}{s}$$

$$c_{p,air}(-100ºC\text{ to } - 50ºC) = 1{,}007\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}0971\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$19{,}7627 \cdot 1{,}007 \cdot (T_3 - 543{,}3377) = 0{,}0971 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{4157{,}592 + 10812{,}9727}{19{,}9010} =$$

$$752{,}2513\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 750$ K; $h_- = 767{,}29$ kJ/kg

$T_+ = 760$ K; $h_+ = 778{,}18$ kJ/kg

$$h_3 = 767{,}29 + (778{,}18 - 767{,}29) \cdot \frac{(752{,}2513 - 750)}{(760 - 750)} = 769{,}7417\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 4{,}8325 \cdot 10^5 \ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 1{,}933 \cdot 10^4 \ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot \left(\frac{P_4}{P_3}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = 769{,}7417 \cdot \left(\frac{1}{25}\right)^{\left(\frac{1{,}4-1}{1{,}4}\right)} = 299{,}8828 \ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 290$ K; $h_- = 290{,}16$ kJ/kg

$T_+ = 300$ K; $h_+ = 300{,}19$ kJ/kg

$$h_4 = 290{,}16 + (300{,}19 - 290{,}16) \cdot \frac{(299{,}8828 - 290)}{(300 - 290)} = 300{,}0724 \ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 19{,}7627 \cdot (769{,}7417 - 300{,}0724) = 9281{,}9214 \ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 295{,}1 = 206{,}57 \ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2066 \ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2066}{0{,}0971} = 2{,}1265 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{9281,9214}{0,0971} = 95552,0018\frac{kJ}{kg}$$

g. Seventh simulation

As the temperature T_1 is known (T_1 = 216,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 210 K; h_- = 209,97 kJ/kg

T_+ = 220 K; h_+ = 219,97 kJ/kg

$$h_1 = 209,97 + (219,97 - 209,97) \cdot \frac{(216,6-210)}{(220-210)} = 216,57\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 14000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0,0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0,0568 \cdot \frac{1,956 \cdot 10^4}{\sqrt{237,9}} = 72,0921\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{72,0921}{5} = 14,4184\ \frac{\text{kg}}{s}$$

Considering $P_1 = 1,41 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 1,41 \cdot 10^4 = 3,5250 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma-1}{\gamma})} = 216,6 \cdot (25)^{(\frac{1,4-1}{1,4})} = 543,3377\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

T_- = 540 K; h_- = 544,35 kJ/kg

T_+ = 550 K; h_+ = 554,74 kJ/kg

$$h_2 = 544,35 + (554,74 - 544,35) \cdot \frac{(543,3377 - 540)}{(550 - 540)} = 547,8171\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 14{,}4184 \frac{kg}{s}$$

$$c_{p,air}(-100ºC \text{ to } -50ºC) = 1{,}007 \left[\frac{kJ}{kg \cdot K}\right]$$

$$\dot{m}_{fuel} = 0{,}0708 \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 \left[\frac{kJ}{kg}\right]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$14{,}4184 \cdot 1{,}007 \cdot (T_3 - 543{,}3377) = 0{,}0708 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{3034{,}092 + 7888{,}9125}{14{,}5193} =$$

$$752{,}3068\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

T_- = 750 K; h_- = 767,29 kJ/kg

T_+ = 760 K; h_+ = 778,18 kJ/kg

$$h_3 = 767{,}29 + (778{,}18 - 767{,}29) \cdot \frac{(752{,}3068 - 750)}{(760 - 750)} = 769{,}8022 \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 3{,}525 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 1{,}41 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 752,3068 \cdot (\frac{1}{25})^{(\frac{1,4-1}{1,4})} = 299,9049\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 290$ K; $h_- = 290,16$ kJ/kg

$T_+ = 300$ K; $h_+ = 300,19$ kJ/kg

$$h_4 = 290,16 + (300,19 - 290,16) \cdot \frac{(299,9049 - 290)}{(300 - 290)} = 300,0946\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 14,4184 \cdot (769,8022 - 300,0946) = 6772,4424\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0,7 \cdot 295,1 = 206,57\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0,2066\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0,2066}{0,0709} = 2,9139 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{6772,4424}{0,0708} = 95534,5235 \frac{kJ}{kg}$$

h. Eighth simulation

As the temperature T_1 is known (T_1 = 216,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of “Ideal gas properties of the air”.

$T_- = 210$ K; $h_- = 209{,}97$ kJ/kg

$T_+ = 220$ K; $h_+ = 219{,}97$ kJ/kg

$$h_1 = 209{,}97 + (219{,}97 - 209{,}97) \cdot \frac{(216{,}6 - 210)}{(220 - 210)} = 216{,}57\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 15000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{1{,}671 \cdot 10^4}{\sqrt{237{,}9}} = 61{,}5879\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{61{,}5879}{5} = 12{,}3176\ \frac{\text{kg}}{s}$$

Considering $P_1 = 1{,}204 \cdot 10^4$ Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 1{,}204 \cdot 10^4 = 3{,}01 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma - 1}{\gamma}\right)} = 216{,}6 \cdot (25)^{\left(\frac{1{,}4 - 1}{1{,}4}\right)} = 543{,}3377\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

$T_- = 540$ K; $h_- = 544{,}35$ kJ/kg

$T_+ = 550$ K; $h_+ = 554{,}74$ kJ/kg

$$h_2 = 544{,}35 + (554{,}74 - 544{,}35) \cdot \frac{(543{,}3377 - 540)}{(550 - 540)} = 547{,}8179\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 12{,}3176\ \frac{kg}{s}$$

$$c_{p,air}(-100ºC \text{ to } -50ºC) = 1{,}007\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}0605\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$12{,}3176 \cdot 1{,}007 \cdot (T_3 - 543{,}3377) = 0{,}0605 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{2591{,}54+6739{,}4544}{12{,}4038} =$$

$$752{,}2688\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 750$ K; $h_- = 767{,}29$ kJ/kg

$T_+ = 760$ K; $h_+ = 778{,}18$ kJ/kg

$$h_3 = 767{,}29 + (778{,}18 - 767{,}29) \cdot \frac{(752{,}2688 - 750)}{(760 - 750)} = 769{,}7608\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 3{,}01 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 1{,}204 \cdot 10^4\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 752{,}2688 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 299{,}8897\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 290$ K; $h_- = 290{,}16$ kJ/kg

$T_+ = 300$ K; $h_+ = 300{,}19$ kJ/kg

$$h_4 = 290{,}16 + (300{,}19 - 290{,}16) \cdot \frac{(299{,}8897 - 290)}{(300 - 290)} = 300{,}0794 \ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 12{,}3176 \cdot (769{,}7608 - 300{,}0794) = 5785{,}3373 \ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 295{,}1 = 206{,}57 \ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2066 \ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2066}{0{,}06055} = 3{,}4116 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{5785{,}3373}{0{,}0605} = 95546{,}4465 \frac{kJ}{kg}$$

i. Ninth simulation

As the temperature T_1 is known (T_1 = 216,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of “Ideal gas properties of the air”.

$T_- = 210$ K; $h_- = 209{,}97$ kJ/kg

$T_+ = 220$ K; $h_+ = 219{,}97$ kJ/kg

$$h_1 = 209{,}97 + (219{,}97 - 209{,}97) \cdot \frac{(216{,}6 - 210)}{(220 - 210)} = 216{,}57 \ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 17000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{1{,}219 \cdot 10^4}{\sqrt{237{,}9}} = 44{,}9286\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{44{,}9286}{5} = 8{,}9857\ \frac{\mathrm{kg}}{s}$$

Considering P_1 = 8787 Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 8787 = 2{,}1967 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = 216{,}6 \cdot (25)^{\left(\frac{1{,}4-1}{1{,}4}\right)} = 543{,}3377\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

T_- = 540 K; h_- = 544,35 kJ/kg

T_+ = 550 K; h_+ = 554,74 kJ/kg

$$h_2 = 544{,}35 + (554{,}74 - 544{,}35) \cdot \frac{(543{,}3377 - 540)}{(550 - 540)} = 547{,}8179\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 8{,}9857\ \frac{kg}{s}$$

$$c_{p,air}(-100^{\underline{o}}\mathrm{C\ to} - 50^{\underline{o}}\mathrm{C}) = 1{,}007\ \left[\frac{kJ}{kg \cdot K}\right]$$

$$\dot{m}_{fuel} = 0{,}04413\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$8{,}9857 \cdot 1{,}007 \cdot (T_3 - 543{,}3377) = 0{,}04413 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{1888{,}764 + 4916{,}4542}{9{,}0486} = 752{,}0729\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 750$ K; $h_- = 767{,}29$ kJ/kg

$T_+ = 760$ K; $h_+ = 778{,}18$ kJ/kg

$$h_3 = 767{,}29 + (778{,}18 - 767{,}29) \cdot \frac{(752{,}0729 - 750)}{(760 - 750)} = 769{,}5474\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 2{,}1967 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 8787\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 752{,}0729 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 299{,}8116\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 290$ K; $h_- = 290{,}16$ kJ/kg

$T_+ = 300$ K; $h_+ = 300{,}19$ kJ/kg

$$h_4 = 290{,}16 + (300{,}19 - 290{,}16) \cdot \frac{(299{,}8116 - 290)}{(300 - 290)} = 300{,}0011\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 8{,}9857 \cdot (769{,}5474 - 300{,}0011) = 4219{,}2093\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 295{,}1 = 206{,}57\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2066\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2066}{0{,}0441} = 4{,}6809 \frac{km}{kg}$$

Finally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{4219{,}2093}{0{,}0441} = 95608{,}6398 \frac{kJ}{kg}$$

j. Tenth simulation

As the temperature T_1 is known (T_1 = 216,6 K), it is possible to compute the enthalpy corresponding to this air state. To do so, it is necessary to interpolate among the values present within the table of "Ideal gas properties of the air".

T_- = 210 K; h_- = 209,97 kJ/kg

T_+ = 220 K; h_+ = 219,97 kJ/kg

$$h_1 = 209{,}97 + (219{,}97 - 209{,}97) \cdot \frac{(216{,}6 - 210)}{(220 - 210)} = 216{,}57\ \frac{kJ}{kg}$$

In order to compute the air mass flow at the inlet, the particular stagnation temperature and pressure of the air at 20000 m above the mean sea level are inserted in the Equation 18:

$$\dot{m}_{inlet} = 0{,}0568 \cdot \frac{P_t}{\sqrt{T_t}} = 0{,}0568 \cdot \frac{7594}{\sqrt{237{,}9}} = 27{,}9891\ \frac{kg}{s}$$

Pondering the bypass ratio, the air mass flow entering the gas generator is:

$$\dot{m}_{core} = \frac{1}{5} \cdot \dot{m}_{inlet} = \frac{27{,}9891}{5} = 5{,}5978\ \frac{\mathrm{kg}}{s}$$

Considering P_1 = 5475 Pa, and knowing the compressor pressure ratio:

$$P_2 = 25 \cdot P_1 = 25 \cdot 5475 = 1{,}3687 \cdot 10^5\ Pa$$

After having undergone an isentropic compression, the temperature T_2 is:

$$T_2 = T_1 \cdot (\frac{P_2}{P_1})^{(\frac{\gamma-1}{\gamma})} = 216{,}6 \cdot (25)^{(\frac{1{,}4-1}{1{,}4})} = 543{,}3377\ K$$

In order to compute the enthalpy corresponding to T_2, it is necessary to interpolate between the values displayed below:

T_- = 540 K; h_- = 544,35 kJ/kg

T_+ = 550 K; h_+ = 554,74 kJ/kg

$$h_2 = 544{,}35 + (554{,}74 - 544{,}35) \cdot \frac{(543{,}3377 - 540)}{(550 - 540)} = 547{,}8179\ \frac{kJ}{kg}$$

In order to obtain the value of T_3, the Equation 20 must be applied. To do so it is necessary to compute several elements.

$$\dot{m}_{core} = 5{,}5978 \frac{kg}{s}$$

$$c_{p,air}(-100ºC \text{ to } -50ºC) = 1{,}007\ [\frac{kJ}{kg \cdot K}]$$

$$\dot{m}_{fuel} = 0{,}0275\ \frac{kg}{s}$$

$$\Delta H_c^o = 42{,}8 \cdot 10^3 [\frac{kJ}{kg}]$$

Considering the above presented characters, and the known value of T_2:

$$\dot{m}_{core} \cdot c_{p,air} \cdot \Delta T_{air} = \dot{m}_{fuel} \cdot \Delta H_c^o;$$

$$5{,}5978 \cdot 1{,}007 \cdot (T_3 - 543{,}3377) = 0{,}0275 \cdot (42{,}8 \cdot 10^3);\ T_3 = \frac{1177{,}856+3062{,}8017}{5{,}6370} =$$

$$752{,}2881\ K$$

The enthalpy corresponding to T_3 is obtained by interpolating between these values:

$T_- = 750$ K; $h_- = 767{,}29$ kJ/kg

$T_+ = 760$ K; $h_+ = 778{,}18$ kJ/kg

$$h_3 = 767{,}29 + (778{,}18 - 767{,}29) \cdot \frac{(752{,}2881 - 750)}{(760 - 750)} = 769{,}7818\ \frac{kJ}{kg}$$

The pressure of the air in the third stage of the cycle is the same as at the second one:

$$P_3 = P_2 = 1{,}3687 \cdot 10^5\ Pa$$

The pressure at the fourth and last stage of the cycle is:

$$P_4 = P_1 = 5475\ Pa$$

In order to define the fourth state of the air, after passing through the turbine set, the Equation 19 is applied again:

$$T_4 = T_3 \cdot (\frac{P_4}{P_3})^{(\frac{\gamma-1}{\gamma})} = 752{,}2881 \cdot (\frac{1}{25})^{(\frac{1{,}4-1}{1{,}4})} = 299{,}8975\ K$$

The enthalpy corresponding to the temperature T_4 is obtained interpolating between the values displayed below:

$T_- = 290$ K; $h_- = 290{,}16$ kJ/kg

$T_+ = 300$ K; $h_+ = 300{,}19$ kJ/kg

$$h_4 = 290{,}16 + (300{,}19 - 290{,}16) \cdot \frac{(299{,}8975 - 290)}{(300 - 290)} = 300{,}0872\ \frac{kJ}{kg}$$

The power outputted by the turbine along the cycle is:

$$\dot{W}_{out} = \dot{m}_{air} \cdot (h_3 - h_4) = 5{,}5978 \cdot (6769{,}7818 - 300{,}0872) = 2629{,}2697\ kW$$

To obtain the ratio of kilometres covered by the propulsion system per each unit mass of fuel consumed, it is necessary to compute the cruising speed:

$$v = M \cdot a = 0{,}7 \cdot 295{,}1 = 206{,}57\ \frac{m}{s}$$

Converting it into a more handy units:

$$v = 0{,}2066\ \frac{km}{s}$$

Now, considering the fuel consumption already mentioned:

$$DCr = \frac{0{,}2066}{0{,}0275} = 7{,}5062\,\frac{km}{kg}$$

inally, to find the ratio of energy or work generated along the cycle per unit mass of fuel consumed:

$$WGr = \frac{2629{,}2697}{0{,}0275} = 95540{,}3232\,\frac{kJ}{kg}$$

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NORMATIVA DEL TFG DE L'EPSEM
Emplenar per l'estudiant/a

ANNEX 1.- AUTORITZACIÓ DE LA MATRÍCULA DEL TFG EN MODALITAT A o B

ESTUDIANT/A: Arnau Coiduras Huguet **Núm. Identificatiu** 47931784Z
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REALITZACIÓ EN ANGLÈS: Sí ☒ No ☐
MODALITAT: A ☐ B ☐

TÍTOL: Aircraft Propulsion Systems: Study And Simulation Of A Turbofan Engine

DESCRIPCIÓ:

This thesis is aimed to the study and comprehension of the behaviour of a well defined turbofan engine under specific operating conditions. Using an interactive programmed block-system within the interface of the software MATLAB Simulink, the performance of a turbofan engine is simulated. Moreover, a deeper study regarding the engine's efficiency at different altitudes is conducted out from a set of characteristic values outputted at each simulation.

SOL·LICITUD PER A SER AVALUAT DE LES COMPETÈNCIES GENÈRIQUES EN NIVELL 3 (si no estan assolides)	Sí	No
1.- Emprenedoria i innovació		X
2.- Sostenibilitat i compromís social		X
3.- Tercera Llengua		X
4.- Comunicació eficaç oral i escrita		X
5.- Ús solvent dels recursos d'informació		X
6.- Aprenentatge autònom		X
7.- Treball en equip		X

Director/a — Signatura

Co-director/a (si s'escau) — Signatura

Estudiant/a — Signatura

Data Registre : ____________________ (validesa: un any des de la data del registre)

3 Exemplars: Director/a / Secretaria de l'Escola / Estudiant/a per incorporar al TFG

* Normativa Transitòria fins que l'aplicatiu PRISMA permeti la gestió telemàtica del TFG
(Document aprovat per la Comissió Permanent de 5 de juliol de 2012, i per la Junta de Centre de 12 de juliol de 2012)

NORMATIVA DEL TFG DE L'EPSEM
Emplenar per l'estudiant/a

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ANNEX 2.- LLIURAMENT DEL TFG

ESTUDIANT/A: Arnau Coiduras Huguet **Núm. Identificatiu** 47931784z
GRAU EN: Enginyeria Mecánica
DIRECTOR/A DEL TFG: Jordi Vives
BREU RESUM DEL TFG (màxim 500 paraules):

This thesis is aimed to the study and comprehension of the behaviour of a well defined turbofan engine under specific operating conditions. Using an interactive programmed block-system within the interface of the software MATLAB Simulink, the performance of a turbofan engine is simulated. Moreover, a deeper study regarding the engine's efficiency at different altitudes is conducted out from a set of characteristic values outputted at each simulation.

PARAULES CLAU (entre 2 i 5):

Turbofan; Aircraft; Propulsion; Efficiency; Altitude

AUTORITZO A PUBLICAR EL TREBALL A UPCommons: Sí No

Director/a	Co-director/a (si s'escau)	Estudiant/a
Signatura	Signatura	Signatura

Data lliurament : 2nd July 2015

Lliurar a la secretaria: 1 còpia en paper de la memòria del TFG.
4 còpies en versió digital (CD o DVD) de la memòria del TFG

2 Exemplars: Secretaria de l'Escola / Estudiant/a

* Normativa Transitòria fins que l'aplicatiu PRISMA permeti la gestió telemàtica del TFG
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